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A NEW MATHEMATICAL MODEL OF CIGARETTE COMBUSTION

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ABSTRACT

The global cigarette combustion problem, which is one of the hardest scientific problems, until now is not fully solved, because its solution is connected with numerous technical and scientific difficulties. For better understanding of this problem, it should be considered as an open multidisciplinary problem. In the offered research work, this problem is treated only from the mathematical point of view, for whom is given a completely new approach toward its solution. It is considered as a complex mathematical problem, which is composed from the following few subproblems: the problem of balancing chemical reaction of tobacco combustion, field temperature problem in the combustion zone, smoke filtration problem, and the problem of groups' formation of reaction coefficients. For solution of these problems were used theories of ordinary and partial differential equations as well as theory of linear vector spaces and theory of groups. These problems are particularly solved for the certain simulation conditions. For instance, the smoke infiltration problem is founded and solved by virtue of partial differential equation of first order. The field temperature problem in the combustion zone is modeled by the two-dimensional heat transfer equation which is solved by quadratures. The chemical reaction which describes tobacco combustion is a completely new reaction and it includes all important alkaloids and toxins. This reaction belongs to the class of continuum chemical reactions with integer oxidation numbers. In fact, it is a very hard chemical reaction, which cannot be balanced by a computer, because right now in the theory of computer sciences there is not powerful software, which can be used for its balance. The unique way to balance this reaction is by the usage of mathematical method. For that particular case we chose a new algebraic method developed by the author. Since, the reaction of tobacco combustion is very complicated we found only its general solution and one particular solution. This reaction spans real vector spaces. For the reaction coefficients are calculated a symmetric group S_{49}, an alternating group A_{49} and 38 primitive groups.

Keywords: *cigarette, cigarette combustion, smoke filtration, field temperature, groups of reaction coefficients.*

НОВ МАТЕМАТИЧКИ МОДЕЛ ЗА ГОРЕЊЕТО НА ЦИГАРАТА

Глобалниот проблем на горење на цигарата, кој е еден од најтешките научни проблеми, до сега не е решен целосно, бидејќи неговото решение е поврзано со низа технички и научни тешкотии. За подобро разбирање на овој проблем, тој би требало да се разгледува како отворен мулти-дисциплинарен проблем. Во предложенава истражувачка работа, овој проблем е третиран само од математичка гледна точка, за кого е даден комплетно нов пристап во правец на негово решавање. Тој е разгледан како комплексен математички проблем, кој е составен од следниве неколку подпроблеми: проблемот на изедначување на хемиската реакција за горење на тутунот, проблемот за температурното поле во зоната на горење, проблемот за филтрација на чадот и проблемот на формирање на групите од коефициентите на реакцијата. За решавање на овие проблеми беа употребени теориите на обични и парцијални диференцијални равенки, како и теоријата на линеарните векторски простори и теоријата на групи. Овие проблеми се партикуларно решени за извесни симулациони услови. На пример, проблемот за филтрација на чадот е заснован и решен врз основа на парцијални диференцијални равенки од прв ред. Проблемот за температурното поле во зоната на горење е моделиран со две-димензионалната равенка за пренос на топлина која е решена со квадратури. Хемиската равенка што го опишува горењето на тутунот е комплетно нова реакција и таа ги вклучува сите битни алкалоиди и токсини. Оваа реакција спаѓа во класата на континуум хемиски реакции со цели оксидациони боеви. Всушност, таа е многу тешка хемиска реакција, која не може да биде изедначена со компјутер, бидејќи сега во теоријата на информатиката нема моќен програм, што може да биде употребен за нејино изедначување. Единствен начин да се изедначи оваа реакција е употреба на математички метод. За овој партикуларен случај избравме нов алгебарски метод развиен од авторот. Бидејќи реакцијата за горење на тутунот е многу комплицирана најдовме само нејзино општо решение и едно партикуларно решение. Оваа реакција разапнува реален векторски простор. За коефициентите на реакцијата се пресметани симетричната група S_{49}, алтернативната група A_{49} и 38 примитивни групи.

Клучни зборови: *цигара, горење на цигарата, филтрација на чадот, температурно поле, групи од коефициентите на реакцијата.*

1. INTRODUCTION

Cigarette smoking as a bad habit with dangerous consequences to people health is always in the focus of scientific research. This topic in professional literature, for instance in medicine, chemistry, tobacco science and so on, is considered from different points of view, but unfortunately in mathematics this problem was totally neglected. Why? It is hard to reply! Perhaps one of the main causes is that this problem looks for multidisciplinary treatment, because its formulation depends of other factors which are out of the mathematics sphere.

Really for its formulation is necessary a strong knowledge of contemporary chemical engineering, heat transfer theory and chemical thermodynamics, while its solution belongs only in the sphere of mathematics. Just this, was author's main challenge and motive to solve this problem in this article.

With a goal to shed light on this important subject, this work will introduce a new approach toward solution of cigarette smoking problem by virtue of mathematical research. With an intention for better understanding of this approach, emphasis is made throughout the prism of theory of balancing chemical reaction of tobacco combustion, field temperature in the combustion zone, smoke filtration problem, and the problem of groups' formation of reaction coefficients.

Now, this question arises: how stay things about this topic in scientific literature? Most of published papers deal with cigarette properties [1-11]. One group of publications is written by authors associated with the tobacco industry. They contain substantial information for burn rate and temperature of the burning cigarette. Among the variables discussed are tobacco type, cigarette dimensions and packing density, filter parameters and paper porosity, and additives. The major objective these publications seems to be to obtain basic understanding of the burning cigarette, with emphasis on reduction of tar, nicotine, carbon monoxide, and other smoke components. Some of these papers have two major limitations for the present objective:

a) most of the data are obtained during the puff, and

b) the results are obtained with the cigarette held in air.

In the following, some basic cigarette characteristics will be discussed first.

2. CIGARETTE CHARACTERISTICS

As a support in understanding the general trend of this work, some of the important factors which the author thinks are interconnected.

• *Cigarette length*. The tobacco column length involves the time a cigarette burns and thus the probability of it being dropped (since it takes some aware effort to light a cigarette, one may suppose that a short burn time reduces the probability that the smoker becomes inattentive and drops the cigarette stub.).

• *Burn rate*. This factor is presented as a change in length or mass with time. Whether a cigarette is burning in air or is being puffed, the burn rate involves the remaining cigarette length and thus the probability of dropping the cigarette.

• *Packing density*. Lower packing density (achieved primarily by the use of expanded tobacco but also involved by the tobacco blend and cut width, *i. e.*, the width of the tobacco strands) reduces the mass of available fuel.

• *Tobacco type*. The tobaccos used in various cigarette packing may vary in heat yield and burn rate due to variations in tobacco blend constituents and ratios, as well as in types and concentrations of flavorings and humectants.

• *Paper parameters*. Paper parameters cause differences in cigarette heat yield and burn rate. The paper permeability affects the flow of oxygen from the outside air to the combustion zone and the diffusion of pyrolysis gases from this zone to the outside. Chemicals are added to the cigarette paper as smolder accelerants or retardants and to modify the appearance of the ash.

• *Filter characteristics*. The presence and nature of filter tips also affects the flow of air through the cigarette. Additional perforations is often provided in the paper covering the filter, reducing the flow of air through the tobacco column (ventilation; this is used to reduce the exposure of the smoker to smoke components).

3. A NEW CHEMICAL FORMAL SYSTEM

In this section we shall develop a new chemical formal system founded by virtue of principles of the theory of real finite dimensional vector spaces [12, 13] and group theory [14].

Into a mathematical model must be introduced a whole set of auxiliary definitions to make the chemistry work consistently. Just this kind of set will be constructed below.

Only on this way chemistry will be consistent and resistant to paradoxes appearance.

Here, by $\mathbb{R}$ is denoted the set of real numbers and by $\mathbb{R}^n$ is denoted the Euclidian n-dimensional vector space with real entries. Throughout, the set of $m\times n$ matrices over a field will be denoted by $\mathbb{R}^{m\times n}$.

Definition 3. 1. *A vector space over the field* $\mathbb{R}$ *consists of a nonempty set V of objects called vectors for which hold the axioms for vector addition*

(A_1) *If* $\boldsymbol{u}, \boldsymbol{v} \in V$, *then* $(\boldsymbol{u} + \boldsymbol{v}) \in V$,

(A_2) $\boldsymbol{u} + \boldsymbol{v} = \boldsymbol{v} + \boldsymbol{u}, \forall \boldsymbol{u}, \boldsymbol{v} \in V$,

(A_3) $\boldsymbol{u} + (\boldsymbol{v} + \boldsymbol{w}) = (\boldsymbol{u} + \boldsymbol{v}) + \boldsymbol{w}, \forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in V$,

(A_4) $\boldsymbol{u} + \boldsymbol{0} = \boldsymbol{u} = \boldsymbol{0} + \boldsymbol{u}, \forall \boldsymbol{u} \in V$,

(A_5) $- \boldsymbol{u} + \boldsymbol{u} = \boldsymbol{0} = \boldsymbol{u} + (- \boldsymbol{u}), \forall \boldsymbol{u} \in V$,

and the axioms for scalar multiplication

(S_1) *If* $\boldsymbol{u} \in V$, *then* $a\boldsymbol{u} \in V, \forall a \in \mathbb{R}$,

(S_2) $a(\boldsymbol{u} + \boldsymbol{v}) = a\boldsymbol{u} + a\boldsymbol{v}, \forall \boldsymbol{u}, \boldsymbol{v} \in V \wedge \forall a \in \mathbb{R}$,

(S_3) $(a + b)\boldsymbol{u} = a\boldsymbol{u} + b\boldsymbol{u}, \forall \boldsymbol{u} \in V \wedge \forall a,b \in \mathbb{R}$,

(S_4) $a(b\boldsymbol{u}) = (ab\boldsymbol{u}), \forall \boldsymbol{u} \in V \wedge \forall a,b \in \mathbb{R}$,

(S_5) $1\boldsymbol{u} = \boldsymbol{u}, \forall \boldsymbol{u} \in V$.

Remark 3. 2. *The content of axioms* (A_1) *and* (S_1) *is described by saying that V is closed under vector addition and scalar multiplication. The element* **0** *in axiom* A_4 *is called the zero vector.*

Definition 3. 3. *If V is a vector space over the field* $\mathbb{R}$, *a subset U of V is called a subspace of V if U is itself a vector space over* $\mathbb{R}$, *where U uses the vector addition and scalar multiplication of V.*

Definition 3. 4. *Let V be a vector space over the field* $\mathbb{R}$, *and let* $\boldsymbol{v}_i \in V$, $(1 \leq i \leq n)$. *Any vector in V of the form*

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n,$$

where $a_i \in \mathbb{R}$, $(1 \leq i \leq n)$ *is called linear combination of* $\boldsymbol{v}_i$, $(1 \leq i \leq n)$.

Definition 3. 5. *The vectors* $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ *are said to span or generate V or are said to form a spanning set of V if* $V = span\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$. *Alternatively,* $\boldsymbol{v}_i \in V$, $(1 \leq i \leq n)$ *span V, if for every vector* $\boldsymbol{v} \in V$, *there exist scalars* $a_i \in \mathbb{R}$, $(1 \leq i \leq n)$ *such that*

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n,$$

i. e., $\boldsymbol{v}$ *is a linear combination of*

$$a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n.$$

Remark 3. 6. *If* $V = \text{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$, *then each vector* $\boldsymbol{v} \in V$ *can be written as a linear combination of the vectors* $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$. *Spanning sets have the property that each vector in V has exactly one representation as a linear combinations of these vectors.*

Definition 3. 7. *Let V be a vector space over a field* $\mathbb{R}$. *The vectors* $\boldsymbol{v}_i \in V$, $(1 \leq i \leq n)$ *are said to be linearly independent over* $\mathbb{R}$, *or simply independent, if it is satisfies the following condition if*

$$s_1\boldsymbol{v}_1 + s_2\boldsymbol{v}_2 + \cdots + s_n\boldsymbol{v}_n = \boldsymbol{0},$$

then

$$s_1 = s_2 = \cdots = s_n = 0.$$

Otherwise, the vectors that are not linearly independent is said to be linearly dependent, or simply dependent.

Remark 3. 8. *The trivial linear combination of the vectors* $\boldsymbol{v}_i$, $(1 \leq i \leq n)$ *is the one with every coefficient zero*

$$0\boldsymbol{v}_1 + 0\boldsymbol{v}_2 + \cdots + 0\boldsymbol{v}_n.$$

Definition 3. 9. *A set of vectors* $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n\}$ *is called a basis of V if it satisfies the following two conditions*

1° $\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n$ *are linearly independent,*

2° $V = span\{\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n\}$.

Definition 3. 10. *A vector space V is said to be of finite dimension n or to be n-dimensional, written dimV = n, if V contains a basis with n elements.*

Definition 3. 11. *The vector space* $\{\boldsymbol{0}\}$ *is defined to have dimension* 0.

Definition 3. 12. *For any matrix* $\boldsymbol{A} \in \mathbb{R}^{m\times n}$ *we denote*

$$Im\boldsymbol{A} = \{\boldsymbol{y} \in \mathbb{R}^m: \boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} \text{ for some } \boldsymbol{x} \in \mathbb{R}^n\}$$

the image of $\boldsymbol{A}$ *or range of* $\boldsymbol{A}$.

Definition 3. 13. *For any matrix* $\boldsymbol{A} \in \mathbb{R}^{m\times n}$ *we denote*

$$Ker\boldsymbol{A} = \{\boldsymbol{x} \in \mathbb{R}^n: \boldsymbol{Ax} = \boldsymbol{0}\}$$

the kernel of $\boldsymbol{A}$ *or null space of* $\boldsymbol{A}$.

Definition 3. 14. *If U and W are subspaces of a vector space V the sum*

$$U + W = \{\boldsymbol{u} + \boldsymbol{w}: \boldsymbol{u} \in U, \boldsymbol{w} \in W\}.$$

Definition 3. 15. *The vector space V is said to be direct sum of its subspaces U and W, denoted by*

$$V = U \oplus W,$$

if every vector $\boldsymbol{v} \in V$ *can be written in one and only one way as* $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w}$, *where* $\boldsymbol{u} \in U$ *and* $\boldsymbol{w} \in W$.

Definition 3. 16. *Let V and U be vector spaces over the field* $\mathbb{R}$. *A mapping* $F: V \to U$ *is called a linear mapping* (*or linear transformation or vector space homomorphism*) *if it satisfies the following two conditions*

1º $\forall \boldsymbol{u}, \boldsymbol{v} \in V, F(\boldsymbol{u} + \boldsymbol{v}) = F(\boldsymbol{u}) + F(\boldsymbol{v})$,

2º $\forall k \in \mathbb{R}, \forall \boldsymbol{u} \in V, F(k\boldsymbol{u}) = kF(\boldsymbol{u})$.

Definition 3. 17. *Let* $F: V \to U$ *be a linear mapping. The kernel of F, written KerF, is the set of elements in V which map into*

$$\boldsymbol{0} \in U: KerF = \{\boldsymbol{v} \in V: F(\boldsymbol{v}) = \boldsymbol{0}\}.$$

Definition 3. 18. *Let* $F: V \to U$ *be a linear mapping. The image of F, written ImF, is the set of image points in U*:

$$ImF = \{\boldsymbol{u} \in U: \exists \boldsymbol{v} \in V \text{ for which } F(\boldsymbol{v}) = \boldsymbol{u}\}.$$

Definition 3. 19. *The rank of a linear map* $F: V \to U$ *is defined to be the dimension of its image, i. e.,*

$$rankF = dim(ImF).$$

Definition 3. 20. *The nullity of a linear map* $F: V \to U$ *is defined to be the dimension of its kernel, i. e.,*

$$nullityF = dim(KerF).$$

Definition 3. 21. *A linear mapping* $F: V \to U$ *is said to be singular if the image of some nonzero vector under F is* $\boldsymbol{0}$, *i. e., if there exists* $\boldsymbol{v} \in V$ *for which* $\boldsymbol{v} \neq \boldsymbol{0}$ *but* $F(\boldsymbol{v}) = \boldsymbol{0}$. *Thus* $F: V \to U$ *is nonsingular if only* $\boldsymbol{0} \in V$ *maps into* $\boldsymbol{0} \in U$ *or equivalently, if its kernel consists only of the zero vector,* $KerF = \{\boldsymbol{0}\}$.

Definition 3. 22. *A mapping* $F: V \to U$ *is called an isomorphism if F is linear and if F is bijective, i. e., if F is one-to-one and onto.*

Definition 3. 23. *A vector space V is said to be isomorphic to a vector space U, written* $V \simeq U$, *if there is an isomorphism* $F: V \to U$.

Definition 3. 24. *An inner product on a vector space V is a function that assigns a number* $\langle \boldsymbol{u}, \boldsymbol{v}\rangle$ *to every pair* $\boldsymbol{u}, \boldsymbol{v}$ *of vectors in V in such a way that the following axioms are satisfied*

(P_1) $\langle \boldsymbol{u}, \boldsymbol{v}\rangle$ *is a real number,* $\forall \boldsymbol{u}, \boldsymbol{v} \in V$,

(P_2) $\langle \boldsymbol{u}, \boldsymbol{v}\rangle = \langle \boldsymbol{v}, \boldsymbol{u}\rangle, \forall \boldsymbol{u}, \boldsymbol{v} \in V$,

(P_3) $\langle \boldsymbol{u} + \boldsymbol{v}, \boldsymbol{w}\rangle = \langle \boldsymbol{u}, \boldsymbol{w}\rangle + \langle \boldsymbol{v}, \boldsymbol{w}\rangle, \forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in V$,

(P_4) $\langle r\boldsymbol{u}, \boldsymbol{v}\rangle = r\langle \boldsymbol{u}, \boldsymbol{v}\rangle, \forall \boldsymbol{u}, \boldsymbol{v} \in V \wedge \forall r \in \mathbb{R}$,

(P_5) $\langle \boldsymbol{u}, \boldsymbol{u}\rangle > 0, \forall \boldsymbol{u} \neq \boldsymbol{0} \in V$.

A vector space V with an inner product $\langle\ ,\ \rangle$ *will be called an inner product space.*

Remark 3. 25. *A real inner product space* $\mathbb{R}^n$ *with the dot product as inner product* $\langle \boldsymbol{u}, \boldsymbol{v}\rangle = \boldsymbol{u} \cdot \boldsymbol{v}$ *is called a Euclidean space.*

Definition 3. 26. *If* $\langle\ ,\ \rangle$ *is an inner product on a space V, the norm or length* $||\boldsymbol{v}||$ *of a vector* $\boldsymbol{v} \in V$, *is defined by* $||\boldsymbol{v}|| = \langle \boldsymbol{v}, \boldsymbol{v}\rangle^{1/2}$.

Definition 3. 27. *Two vectors* $\boldsymbol{u}$ *and* $\boldsymbol{v}$ *in an inner product space V are said to be orthogonal, written* $\boldsymbol{u} \perp \boldsymbol{v}$, *if* $\langle \boldsymbol{u}, \boldsymbol{v}\rangle = 0$.

Definition 3. 28. *Two vectors* $\boldsymbol{u}$ *and* $\boldsymbol{v}$ *in an inner product space V are said to be orthogonal, written* $\boldsymbol{u} \perp \boldsymbol{v}$, *if* $\langle \boldsymbol{u}, \boldsymbol{v}\rangle = 0$.

Definition 3. 29. *A set* $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \dots, \boldsymbol{e}_n\}$ *of vectors is called an orthogonal set of vectors if each* $\boldsymbol{e}_i \neq \boldsymbol{0}$, $(1 \le i \le n)$ *and* $\langle \boldsymbol{e}_i, \boldsymbol{e}_j\rangle = 0, \forall i \neq j$.

Definition 3. 30. *If, in addition,* $||\boldsymbol{e}_i|| = 1, \forall i$, *the set* $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \dots, \boldsymbol{e}_n\}$ *is called an orthonormal set.*

Definition 3. 31. *Let U be a subspace of an inner product space V. The orthogonal complement* $U^{\perp}$ *of U in V is defined by*

$$U^{\perp} = \{\boldsymbol{v}: \boldsymbol{v} \in V, \langle \boldsymbol{v}, \boldsymbol{u}\rangle = 0, \forall \boldsymbol{u} \in U\}.$$

Definition 3. 32. *Let* $S = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_n\}$ *be a set of vectors in an inner product space V, then S is said to be orthogonal if each of its vectors are nonzero and if its vectors are mutually orthogonal, i. e., if* $\langle \boldsymbol{v}_i, \boldsymbol{v}_i\rangle \neq 0$ *but* $\langle \boldsymbol{v}_i, \boldsymbol{v}_j\rangle = 0, \forall i \neq j$.

Definition 3. 33. *A permutation* σ, *with a notation* $\sigma = (j_1, j_2, \dots, j_n)$, *where* $j_i = \sigma(i)$, $(1 \le i \le n)$ *of a finite set* $\mathfrak{I}$ *is a one-to-one mapping of* $\mathfrak{I}$ *into itself.*

Definition 3. 34. *In the particular case where* $\mathfrak{I} = \{1, 2, \dots, n\}$, *we write* $\mathfrak{I} = S_n$, *then* S_n *is called the symmetric group of degree n.*

Definition 3. 35. *The alternating group of degree n, denoted by* A_n, *is a set of even permutation in* S_n.

Definition 3. 36. *By an inversion in* σ *we mean a pair of integers* (i, k) *such that* $i > k$, *but i precedes k in* σ.

Definition 3. 37. *The sign of* σ, *written* $sgn\sigma$, *is defined by* $sgn\sigma = (-1)^k$, *where k is a total number of inversions in* σ.

Definition 3. 38. *A permutation σ is said to be even or odd according as there is an even or odd total number of inversions in σ.*

Definition 3. 39. *A transposition is a permutation τ which interchanges two numbers, i and j > i, and lives the other numbers fixed*:

$$\tau = (1\ 2\ \dots\ (i - 1)j(i + 1)\ \dots\ (j - 1)i(j + 1)\ \dots\ n).$$

Definition 3. 40. *If a_i, $(1 \le i \le m)$ are distinct integers in S_n, $(a_1, a_2, \dots, a_m)$ stands for the permutation that maps each integer in $S_n - \{a_1, a_2, \dots, a_m\}$ to itself, and maps $a_1 \to a_2$, $a_2 \to a_3, \dots, a_{m-1} \to a_m$, $a_m \to a_1$, we call such a permutation an orbit $\varnothing$ of length m.*

Definition 3. 41. *Let n be a positive integer and σ be a permutation, such that $\sigma^n = \iota$, where ι is an identity permutation, then the permutation σ is of order n.*

Let G be a finite group, of order $|G|$.

Definition 3. 42. *The center of G is the set of elements which commute with all elements of G.*

It is a normal subgroup of G. The center of G equals G if and only if G is abelian.

Definition 3. 43. *Two elements g_1 and g_2 of the group G are conjugate, if there is an element $h \in G$ such that*

$$hg_1h^{-1} = g_2.$$

Definition 3. 44. *The conjugacy class of an element $g \in G$ is the set of elements conjugate to g.*

Definition 3. 45. *In the same way, two subgroups H_1 and H_2 of G are conjugate, if there is an element $h \in G$ such that*

$$hH_1h^{-1} = H_2.$$

Definition 3. 46. *A subgroup H is normal if there is no other subgroup conjugate to it.*

Definition 3. 47. *G is a simple group if it contains no normal subgroup other than G and the trivial subgroup.*

Definition 3. 48. *The commutator subgroup or derived subgroup of G, [G, G], is the subgroup generated by all the commutators $g_1g_2g_1^{-1}g_2^{-1}$.*

It is a normal subgroup of G, the smallest such that the quotient group is abelian. $[G, G]$ is trivial if and only if G is abelian.

Definition 3. 49. *G is a perfect group if $[G, G] = G$.*

Definition 3. 50. *The derived series of G is the series of subgroups $N_1 \supset N_2 \supset \cdots \supset N_k$, where $N_1 = [G, G]$ (commutator subgroup), and $N_i = [N_{i-1}, N_{i-1}]$ for $i > 1$.*

And the series stops at N_k such that $N_k = [N_k, N_k]$. All the terms N_i in the derived series are normal subgroups of G.

Definition 3. 51. *G is a solvable group if the derived series stops at the trivial subgroup.*

Definition 3. 52. *The exponent of G is the lcm of orders of all elements of G. It divides $|G|$.*

Definition 3. 53. *The lower central series of G is the series of subgroups $N_1 \supset N_2 \supset \cdots \supset N_k$, where $N_1 = [G, G]$, and $N_i = [G, N_{i-1}]$ for $i > 1$.*

And the series stops at N_k such that $N_k = [G, N_k]$.

All the terms N_i in the lower central series are normal subgroups of G.

Definition 3. 54. *The normal closure of a subgroup H is the subgroup N of G generated by elements in H and all its conjugates.*

N is a normal subgroup of G.

Definition 3. 55. *The normalizer of a subgroup H is the largest subgroup N of G containing H, such that H is a normal subgroup of N.*

Let p be a prime factor of $|G|$.

Definition 3. 56. *A p-Sylow subgroup of G is a maximal subgroup H whose order is a power of p.*

$|H|$ equals the largest power of p dividing $|G|$.

A p-Sylow subgroup needs not to be normal, but all p-Sylow subgroups are conjugate to each other.

Definition 3. 57. *The upper central series of G is the series of subgroups $N_1 \supset N_2 \supset \cdots \supset N_k$, where N_k is the center of G, and N_i is the center of the quotient group G/N_{i+1} for $i < k$.*

And either $N_1 = G$, or G/N_1 has trivial center. All the terms N_i in the upper central series are normal subgroups of G.

Definition 3. 58. *G is a nilpotent group if $N_1 = G$.*

An essential role for every chemical equation plays its stability. For that goal here is introduced a new criterion for stability of chemical equations.

Let an arbitrary chemical reaction is given in its algebraically free form

$$\sum_{j=1}^{n} x_jM_j \to 0, \tag{3. 1}$$

where x_j, $(1 \le j \le n)$ are required rational coefficients and M_j, $(1 \le j \le n)$ are molecules, then its stability array can be constructed on this manner

$$\begin{array}{cccccc} x_n & x_{n-1} & x_{n-2} & \dots & x_2 & x_1 \\ x_1 & x_2 & x_3 & \dots & x_{n-1} & x_n \\ a_1 & a_2 & a_3 & \dots & a_{n-1} & 0 \\ b_1 & b_2 & b_3 & \dots & 0 & 0 \\ \vdots & & & & & \\ z_1 & z_2 & 0 & \dots & 0 & 0 \end{array}$$

The elements in third rows are calculated as shown below

$$a_1 = (x_1x_{n-1} - x_2x_n)/x_1,$$
$$a_2 = (x_1x_{n-2} - x_3x_n)/x_1,$$
$$a_3 = (x_1x_{n-3} - x_4x_n)/x_1,$$
$$\vdots$$
$$a_{n-1} = (x_1x_1 - x_nx_n)/x_1,$$

while the elements in fourth row are

$$b_1 = (a_1x_2 - x_1a_2)/a_1,$$
$$b_2 = (a_1x_3 - x_1a_3)/a_1,$$
$$b_3 = (a_1x_4 - x_1a_4)/a_1,$$
$$\vdots$$
$$b_{n-2} = (a_1x_{n-1} - x_1a_{n-1})/a_1,$$
$$\vdots$$

and the elements in the last row are

$$z_1 = (q_1p_2 - p_1q_2)/q_1$$

and

$$z_2 = (q_1p_3 - p_1q_3)/q_1.$$

Definition 3. 59. *For chemical reaction* (3. 1) *to be stable the primary requirement is the elements in first column of the above array to have the same sign.*

Other results for stability criteria are obtained in works [15, 16] for some general classes of complex vector functional equations [17-19].

Let $\mathscr{X}$ be a finite set of molecules.

Definition 3. 60. *A chemical reaction on* $\mathscr{X}$ *is a pair of formal linear combinations of elements of* $\mathscr{X}$, *such that*

$$\rho: \sum_{j=1}^{r} a_{ij}x_j \to \sum_{j=1}^{s} b_{ij}y_j, \ (1 \le i \le m) \quad (3.\ 2)$$

with $a_{ij}, b_{ij} \ge 0$.

Definition 3. 61. *Chemical equation is a numerical quantification of a chemical reaction*.

In [20] is proved the following proposition.

Proposition 3. 62. *Any chemical equation may be presented in this algebraic form*

$$\sum_{j=1}^{s} x_j \prod_{i=1}^{m} \Psi^{i}{}_{a_{ij}} = \sum_{j=s+1}^{n} x_j \prod_{i=1}^{m} \Psi^{i}{}_{b_{ij}}, \quad (3.\ 3)$$

where x_j, $(1 \le j \le n)$ *are unknown rational coefficients,* $\Psi^{i}{}_{a_{ij}}$ and $\Psi^{i}{}_{b_{ij}}$, $(1 \le i \le m)$ *are chemical elements in reactants and products, respectively,* a_{ij} *and* b_{ij}, $(1 \le i \le m;\ 1 \le j \le n;\ m < n)$ *are numbers of atoms of elements* $\Psi^{i}{}_{a_{ij}}$ *and* $\Psi^{i}{}_{b_{ij}}$, *respectively, in j-th molecule.*

Definition 3. 63. *Each chemical reaction* ρ *has a domain*

$$Dom\rho = \{x \in \mathscr{X} \mid a_{ij} > 0\}. \quad (3.\ 4)$$

Definition 3. 64. *Each chemical reaction* ρ *has an image*

$$Im\rho = \{y \in \mathscr{X} \mid b_{ij} > 0\}. \quad (3.\ 5)$$

Definition 3. 65. *Chemical reaction* ρ *is generated for some* $x \in \mathscr{X}$, *if both* $a_{ij} > 0$ *and* $b_{ij} > 0$.

Definition 3. 66. *For the case as the previous definition, we say x is a generator* of ρ.

Definition 3. 67. *The set of generators of* ρ *is thus* $Dom\rho \cap Im\rho$.

Often chemical reactions are modeled like pairs of multisets, corresponding to integer stoichiometric constants.

Definition 3. 68. *A stoichiometrical space is a pair* $(\mathscr{X}, \mathscr{R})$, *where* $\mathscr{X}$ *is a set of chemical reactions on* $\mathscr{X}$. *It may be symbolized by an arc-weighted bipartite directed graph* $\Gamma(\mathscr{X}, \mathscr{R})$ *with vertex set* $\mathscr{X} \cup \mathscr{R}$, *arcs* $x \to \rho$ *with weight* a_{ij} *if* $a_{ij} > 0$, *and arcs* $\rho \to y$ *with weight* b_{ij} *if* $b_{ij} > 0$.

Let us now consider an arbitrary subset $\mathscr{A} \subseteq \mathscr{X}$.

Definition 3. 69. *A chemical reaction* ρ *may take place in a reaction combination composed of the molecules in* A *if and only if* $Dom\rho \subseteq \mathscr{A}$.

Definition 3. 70. *The collection of all possible reactions in the stoichiometrical space* $(\mathscr{X}, \mathscr{R})$, *that can start from* $\mathscr{A}$ *is given by*

$$\mathscr{R}_{\mathscr{A}} = \{\rho \in \mathscr{R} \mid Dom\rho \subseteq \mathscr{A}\}. \quad (3.\ 6)$$

4. Reaction of Tobacco Combustion

Begin of this section we shall create with a theoretical approach, *i. e.*, we shall give a completely new method for balancing chemical reactions.

We shall do that, because most of the current chemical ways for balancing chemical reactions are out of order, or they are useless for complex reactions.

Theorem 4. 1. *Any chemical equation may be presented in a free algebraic form on this way*

$$\sum_{j=1}^{n} x_j \prod_{i=1}^{m} \Psi^{i}{}_{a_{ij}} = 0, \quad (4.\ 1)$$

where x_j, $(1 \le j \le n)$ *are unknown rational coefficients,* Ψ^{i}, $(1 \le i \le m)$ *are chemical* *elements and* a_{ij}, $(1 \le i \le m,\ 1 \le j \le n)$ *are atom numbers of i-th element* Ψ^i *in j-th molecule.*

Proof. Let there exists an arbitrary chemical equation from m distinct elements and n molecules

$$\sum_{j=1}^{n} x_j \boldsymbol{v}_j = 0, \qquad (4.\ 2)$$

where

$$\boldsymbol{v}_j = \Psi^1{}_{a_{1j}}\Psi^2{}_{a_{2j}} \cdots \Psi^m{}_{a_{mj}}, \ (1 \le j \le n).$$

Then previous expression becomes

$$\sum_{j=1}^{n} \Psi^1{}_{a_{1j}}\Psi^2{}_{a_{2j}} \cdots \Psi^m{}_{a_{mj}} = 0. \qquad (4.\ 3)$$

If we write the above equation in a compact form, then immediately follows (4. 1). □

Theorem 4. 2. *The chemical equation* (4. 1) *reduces to the following matrix equation*

$$\boldsymbol{Ax} = \boldsymbol{0}, \quad (4.\ 4)$$

where $\boldsymbol{A} = [a_{ij}]_{m\times n}$, $(1 \le i \le m,\ 1 \le j \le n)$ *is a reaction matrix,* $\boldsymbol{x}^{\mathrm{T}} = (x_1, x_2, \ldots, x_n)$ *is a column vector of the coefficients* x_j, $(1 \le j \le n)$ *and* $\boldsymbol{0}^{\mathrm{T}} = (0, 0, \ldots, 0)$ *is a null column vector of order m, and* T *denotes transpose*.

Proof. If we develop the molecules of the reaction (4. 1) in an explicit form, then we obtain the reaction matrix $\boldsymbol{A}$ shown below

	$\boldsymbol{v}_1 = \Psi^1{}_{a_{11}}\Psi^2{}_{a_{21}}\cdots\Psi^m{}_{a_{m1}}$	$\boldsymbol{v}_2 = \Psi^1{}_{a_{12}}\Psi^2{}_{a_{22}}\cdots\Psi^m{}_{a_{m2}}$	...	$\boldsymbol{v}_n = \Psi^1{}_{a_{1n}}\Psi^2{}_{a_{2n}}\cdots\Psi^m{}_{a_{mn}}$
Ψ^1	a_{11}	a_{12}	...	a_{1n}
Ψ^2	a_{21}	a_{22}	...	a_{2n}
$\vdots$				
Ψ^m	a_{m1}	a_{m2}	...	a_{mn}

From the above development we obtain that

$$\boldsymbol{v}_j = \sum_{i=1}^{m} a_{ij}\Psi^i, \ (1 \le j \le n). \qquad (4.\ 5)$$

If we substitute (4. 5) into (4. 2), follows

$$\sum_{j=1}^{n} x_j \sum_{i=1}^{m} a_{ij}\Psi^i = 0, \qquad (4.\ 6)$$

or

$$\sum_{i=1}^{m} \Psi^i \sum_{j=1}^{n} a_{ij}x_j = 0, \qquad (4.\ 7)$$

i.e.,

$$\sum_{j=1}^{n} a_{ij}x_j = 0, \ (1 \le i \le m). \qquad (4.\ 8)$$

Last equation if we present in a matrix form, actually we obtain (4. 4). □

According to [21], the deterministic approach is important, since it enables us to classify the chemical reaction as:

1º *impossible*, when the system (4. 8) is inconsistent.

2º *unique*, (within relative proportions) when the system (4. 8) has unique solution.

3º *non-unique*, when the system (4. 8) has an infinite number of solutions.

Last kind of the reactions exhibit infinite linearly independent solutions all of which satisfy the chemical balance, and yet they are not all chemically feasible solutions for a given set of experimental conditions. A unique solution is obtained by imposing a chemical constraint, namely, that reactants have to react only in certain proportions.

The coefficients satisfy three basic principles

- the law of conservation of atoms,
- the law of conservation of mass, and
- the time-independence of the reaction.

Theorem 4. 3. *Suppose that chemical equation* (4. 2) *is a vector space V over the field* ℝ *spanned by the vectors of the molecules* $\boldsymbol{v}_i$ $(1 \le i \le n)$. *If any set of m vectors of the molecules in V is linearly independent, then m* ≤ *n.*

Proof. Let

$$V = \mathrm{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}.$$

We must show that every set $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_m\}$ of vectors in V with $m > n$ fails to be linearly independent. This is accomplished by showing that numbers $x_1, x_2, \ldots, x_m$ can be found, not all zero, such that

$$\sum_{j=1}^{m} x_j\boldsymbol{u}_j = x_1\boldsymbol{u}_1 + x_2\boldsymbol{u}_2 + \cdots + x_m\boldsymbol{u}_m = \boldsymbol{0}.$$

Since V is spanned by the vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$, each vector $\boldsymbol{u}_j$ can be expressed as a linear combination of $\boldsymbol{v}_i$

$$\boldsymbol{u}_j = a_{1j}\boldsymbol{v}_1 + a_{2j}\boldsymbol{v}_2 + \cdots + a_{nj}\boldsymbol{v}_n = \sum_{i=1}^{n} a_{ij}\boldsymbol{v}_j.$$

Substituting these expressions into the preceding equation gives

$$\boldsymbol{0} = \sum_{j=1}^{m} x_j \Big(\sum_{i=1}^{n} a_{ij}\boldsymbol{v}_j\Big) = \sum_{i=1}^{n} \Big(\sum_{j=1}^{m} a_{ij}x_j\Big)\boldsymbol{v}_i.$$

This is certainly the case if each coefficient of $\boldsymbol{v}_i$ is zero, *i. e.*, if

$$\sum_{j=1}^{m} a_{ij}x_j = 0, \ (1 \le i \le n).$$

But this is a system of n equations in the m variables $x_1, x_2, \ldots, x_m$, so because $m > n$, it has a nontrivial solution. This is what we wanted.□

Now we shall prove the following results.

Theorem 4. 4. *Let U be a subset of a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$. *Then U is a subspace of V if and only if it satisfies the following conditions*

$$\mathbf{0} \in U, \ \mathbf{0} \text{ is the zero vector of } V, \quad (4.\,9)$$

$$\text{If } \boldsymbol{u}_1, \boldsymbol{u}_2 \in U, \text{ then } (\boldsymbol{u}_1 + \boldsymbol{u}_2) \in U, \quad (4.\,10)$$

$$\text{If } \boldsymbol{u} \in U, \text{ then } a\boldsymbol{u} \in U, \ \forall a \in \mathbb{R}. \quad (4.\,11)$$

Proof. If U is a subspace of V of the chemical equation (4. 1), it is clear by axioms (A_1) and (S_1), that the sum of two vectors in U is again in U and that any scalar multiple of a vector in U is again in U. By other words, U is *closed* under the vector addition and scalar multiplication of V. The nice part is that the converse is also true, *i. e.*, if U is closed under these operations, then all the other axioms are automatically satisfied. For instance, axiom (A_2) asserts that holds $\boldsymbol{u}_1 + \boldsymbol{u}_2 = \boldsymbol{u}_2 + \boldsymbol{u}_1$, $\forall \boldsymbol{u}_1, \boldsymbol{u}_2 \in U$. But, this is clear because the equation is already true in V, and U uses the same addition as V. Similarly, axioms (A_3), (S_2), (S_3), (S_4) and (S_5) hold automatically in U, because they are true in V. All that remains is to verify axioms (A_4) and (A_5).

If (4. 9), (4. 10) and (4. 11) hold, then axiom (A_4) follows from (4. 9) and axiom (A_5) follows from (4. 11), because $-\boldsymbol{u} = (-1)\boldsymbol{u}$ lies in U, $\forall \boldsymbol{u} \in U$. Hence U is a subspace by the above discussion. Conversely, if U is a subspace it is closed under addition and scalar multiplication, and this gives (4. 10) and (4. 11). If $\boldsymbol{z}$ denotes the zero vector of U, then $\boldsymbol{z} = 0\boldsymbol{z}$ in U. But, $0\boldsymbol{z} = \mathbf{0}$ in V, so $\mathbf{0} = \boldsymbol{z}$ lies in U. This proves (4. 9). □

Remark 4. 5. *If U is a subspace of V of the chemical equation* (4. 2) *over the field, then the above proof shows that U and V share the same zero vector. Also, if* $\boldsymbol{u} \in U$, *then* $-\boldsymbol{u} = (-1)\boldsymbol{u} \in U$, *i. e., the negative of a vector in U is the same as its negative in V.*

Proposition 4. 6. *If V is any vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$, *then* $\{\mathbf{0}\}$ *and V are subspaces of V.*

Proof. $U = V$ clearly satisfies the conditions of the Theorem 4. 4. As to $U = \{\mathbf{0}\}$, it satisfies the conditions because

$$\mathbf{0} + \mathbf{0} = \mathbf{0} \text{ and } a\mathbf{0} = \mathbf{0}, \ \forall a \in \mathbb{R}. \quad □$$

Remark 4. 7. *The vector space* $\{\mathbf{0}\}$ *is called the zero subspace of V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$. *Since all zero subspaces look alike, we speak of the zero vector space and denote it by* 0. *It is the unique vector space containing just one vector.*

Proposition 4. 8. *If* $\boldsymbol{v}$ *is a vector of some molecule in a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$, *then the set* $\mathbb{R}\boldsymbol{v} = \{a\boldsymbol{v}, \forall a \in \mathbb{R}\}$ *of all scalar multiples of* $\boldsymbol{v}$ *is a subspace of V.*

Proof. Since $\mathbf{0} = 0\boldsymbol{v}$, it is clear that $\mathbf{0}$ lies in $\mathbb{R}\boldsymbol{v}$. Given two vectors $a\boldsymbol{v}$ and $b\boldsymbol{v}$ in $\mathbb{R}\boldsymbol{v}$, their sum $a\boldsymbol{v} + b\boldsymbol{v} = (a + b)\boldsymbol{v}$ is also a scalar multiple of $\boldsymbol{v}$ and so lies in $\mathbb{R}\boldsymbol{v}$. Therefore $\mathbb{R}\boldsymbol{v}$ is closed under addition. Finally, given $a\boldsymbol{v}$, $r(a\boldsymbol{v}) = (ra)\boldsymbol{v}$ lies in $\mathbb{R}\boldsymbol{v}$, so $\mathbb{R}\boldsymbol{v}$ is closed under scalar multiplication. If we take into account the Theorem 4. 4, immediately follows the statement of the proposition. □

Proposition 4. 9. *Let* $\boldsymbol{A} \in \mathbb{R}^{m\times n}$. *The set* $ImgU = \{\boldsymbol{Ax}: \boldsymbol{x} \in \mathbb{R}^n\}$, *called the range or image of the matrix* $\boldsymbol{A}$ *is a subspace of* $\mathbb{R}^m$.

Proof. Note first that U is in fact a subset of $\mathbb{R}^m$, because $\boldsymbol{A}$ is $m\times n$. Each vector in U is of the form $\boldsymbol{Ax}$ for some vector $\boldsymbol{x} \in \mathbb{R}^n$. To apply the Theorem 4. 4, note that $\mathbf{0} = \boldsymbol{A}\mathbf{0}$ has the required form, so $\mathbf{0}$ lies in U. Similarly, the equation

$$\boldsymbol{Ax} + \boldsymbol{Ay} = \boldsymbol{A}(\boldsymbol{x} + \boldsymbol{y})$$

and

$$r(\boldsymbol{Ax}) = \boldsymbol{A}(r\boldsymbol{x})$$

show that sums and scalar multiplies of vector in U again have the required form. Hence U is a subspace of $\mathbb{R}^m$. □

Proposition 4. 10. *Let* $\boldsymbol{A} \in \mathbb{R}^{m\times n}$. *The set*

$$nullU = KerU = \{\boldsymbol{Ax} = \mathbf{0}: \boldsymbol{x} \in \mathbb{R}^n\},$$

called the null space or kernel of the matrix $\boldsymbol{A}$ *is a subspace of* $\mathbb{R}^n$.

Proof. Here U consists of all columns $\boldsymbol{x}$ in $\mathbb{R}^n$ satisfying the condition that $\boldsymbol{Ax} = \mathbf{0}$. Since $\boldsymbol{A}\mathbf{0} = \mathbf{0}$, it is clear that $\mathbf{0}$ lies in U. If $\boldsymbol{x}$ and $\boldsymbol{y}$ both lie in U, then

$$\boldsymbol{A}(\boldsymbol{x} + \boldsymbol{y}) = \boldsymbol{Ax} + \boldsymbol{Ay} = \mathbf{0} + \mathbf{0} = \mathbf{0}.$$

This shows that $\boldsymbol{x} + \boldsymbol{y}$ qualifies for membership in U, so U is closed under addition. Similarly,

$$\boldsymbol{A}(r\boldsymbol{x}) = r(\boldsymbol{Ax}) = r\mathbf{0} = \mathbf{0},$$

so $r\boldsymbol{x}$ lies in U. Thus U is closed under scalar multiplication and is a subspace of $\mathbb{R}^n$. □

Theorem 4. 11. *Let*

$$U = span\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$$

in a vector space V of the chemical equation (4. 2) *over the field* $\mathbb{R}$. *Then,*

U is a subspace of V containing each of $\boldsymbol{v}_i$, $(1 \le i \le n)$, (4. 12)

U is the smallest subspace in the sense that any subspace of V that contains each of $\boldsymbol{v}_i$, $(1 \le i \le n)$ *must contain U.* (4. 13)

Proof. First we shall proof (4. 12).

Clearly

$$\mathbf{0} = 0\boldsymbol{v}_1 + 0\boldsymbol{v}_2 + \cdots + 0\boldsymbol{v}_n$$

belongs to U. If

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n$$

and

$$\boldsymbol{w} = b_1\boldsymbol{v}_1 + b_2\boldsymbol{v}_2 + \cdots + b_n\boldsymbol{v}_n$$

are two members of U and $a \in U$, then

$$\boldsymbol{v} + \boldsymbol{w} = (a_1 + b_1)\boldsymbol{v}_1 + (a_2 + b_2)\boldsymbol{v}_2 + \cdots + (a_n + b_n)\boldsymbol{v}_n,$$

$$a\boldsymbol{v} = (aa_1)\boldsymbol{v}_1 + (aa_2)\boldsymbol{v}_2 + \cdots + (aa_n)\boldsymbol{v}_n,$$

so both $\boldsymbol{v} + \boldsymbol{w}$ and $a\boldsymbol{v}$ lie in U. Hence U is a subspace of V. It contains each of $\boldsymbol{v}_i$, $(1 \le i \le n)$. For instance,

$$\boldsymbol{v}_2 = 0\boldsymbol{v}_1 + 1\boldsymbol{v}_2 + 0\boldsymbol{v}_3 + \cdots + 0\boldsymbol{v}_n.$$

This proves (4. 12).

Now, we shall prove (4. 13). Let W be subspace of V that contains each of $\boldsymbol{v}_i$, $(1 \le i \le n)$. Since W is closed under scalar multiplication, each of $a_i\boldsymbol{v}_i$, $(1 \le i \le n)$ lies in W for any choice of a_i, $(1 \le i \le n)$ in $\mathbb{R}$. But, then $a_i\boldsymbol{v}_i$, $(1 \le i \le n)$ lies in W, because W is closed under addition. This means that W contains every member of U, which proves (4. 13). □

Theorem 4. 12. *The intersection of any number of subspaces of a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *is a subspace of V.*

Proof. Let $\{W_i: i \in \boldsymbol{I}\}$ be a collection of subspaces of V and let $W = \cap\, (W_i: i \in \boldsymbol{I})$. Since each W_i is a subspace, then $\mathbf{0} \in W_i$, $\forall i \in \boldsymbol{I}$. Thus $\mathbf{0} \in W$. Assume $\boldsymbol{u}, \boldsymbol{v} \in W$. Then, $\boldsymbol{u}, \boldsymbol{v} \in W_i$, $\forall i \in \boldsymbol{I}$. Since each W_i is a subspace, then $(a\boldsymbol{u} + b\boldsymbol{v}) \in W_i$, $\forall i \in \boldsymbol{I}$. Therefore $(a\boldsymbol{u} + b\boldsymbol{v}) \in W$. Thus W is a subspace of V of the chemical equation (4. 2). □

Theorem 4. 13. *The union* $W_1 \cup W_2$ *of subspaces of a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *need not be a subspace of V.*

Proof. Let $V = \mathbb{R}^2$ and let $W_1 = \{(a, 0): a \in \mathbb{R}\}$ and $W_2 = \{(0, b): b \in \mathbb{R}\}$. That is, W_1 is the x-axis and W_2 is the y-axis in $\mathbb{R}^2$. Then W_1 and W_2 are subspaces of V of the chemical equation (4. 2). Let $\boldsymbol{u} = (1, 0)$ and $\boldsymbol{v} = (0, 1)$. Then the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ both belong to the union $W_1 \cup W_2$, but $\boldsymbol{u} + \boldsymbol{v} = (1, 1)$ does not belong to $W_1 \cup W_2$. Thus $W_1 \cup W_2$ is not a subspace of V. □

Theorem 4. 14. *The homogeneous system of linear equations* (4. 8), *obtained from the chemical equation* (4. 1), *in n unknowns* $x_1, x_2, \ldots, x_n$ *over the field* $\mathbb{R}$ *has a solution set W, which is a subspace of the vector space* $\mathbb{R}^n$.

Proof. The system (4. 8) is equivalent to the matrix equation (4. 4). Since $\boldsymbol{A}\mathbf{0} = \mathbf{0}$, the zero vector $\mathbf{0} \in W$. Assume $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors in W, *i. e.*, $\boldsymbol{u}$ and $\boldsymbol{v}$ are solutions of the matrix equation (4. 4). Then $\boldsymbol{A}\boldsymbol{u} = \mathbf{0}$ and $\boldsymbol{A}\boldsymbol{v} = \mathbf{0}$.

Therefore, $\forall a, b \in \mathbb{R}$, we have

$$\boldsymbol{A}(a\boldsymbol{u} + b\boldsymbol{v}) = a\boldsymbol{A}\boldsymbol{u} + b\boldsymbol{A}\boldsymbol{v} = a\mathbf{0} + b\mathbf{0} = \mathbf{0} + \mathbf{0} = \mathbf{0}.$$

Hence $a\boldsymbol{u} + b\boldsymbol{v}$ is a solution of the matrix equation (4. 4), *i. e.*, $a\boldsymbol{u} + b\boldsymbol{v} \in W$. Thus W is a subspace of $\mathbb{R}^n$. □

Theorem 4. 15. *If S is a subset of the vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$, *then*

1° *the set span*$\{S\}$ *is a subspace of V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *which contains S.*

2° *span*$\{S\} \subseteq W$, *if W is any subspace of V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *containing S.*

Proof. 1°. If $S = \varnothing$, then span$\{S\} = \{\mathbf{0}\}$, which is a subspace of V containing the empty set $\varnothing$. Now assume $S \ne \varnothing$. If $\boldsymbol{v} \in S$, then $1\boldsymbol{v} = \boldsymbol{v} \in$ span$\{S\}$, therefore S is a subset of span$\{S\}$. Also, span$\{S\} \ne \varnothing$ because $S \ne \varnothing$. Now assume $\boldsymbol{v}, \boldsymbol{w} \in$ span$\{S\}$; say

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + \cdots + a_m\boldsymbol{v}_m$$

and

$$\boldsymbol{v} = b_1\boldsymbol{w}_1 + \cdots + b_n\boldsymbol{w}_n$$

where $\boldsymbol{v}_i, \boldsymbol{w}_j \in S$ and a_i, b_j are scalars.

Then

$$\boldsymbol{v} + \boldsymbol{w} = a_1\boldsymbol{v}_1 + \cdots + a_m\boldsymbol{v}_m + b_1\boldsymbol{w}_1 + \cdots + b_n\boldsymbol{w}_n$$

and for any scalar k,

$$k\boldsymbol{v} = k(a_1\boldsymbol{v}_1 + \cdots + a_m\boldsymbol{v}_m) = ka_1\boldsymbol{v}_1 + \cdots + ka_m\boldsymbol{v}_m$$

belong to span$\{S\}$ because each is a linear combination of vectors in S. Thus span$\{S\}$ is a subspace of V of the chemical equation (4. 2) over the field $\mathbb{R}$ which contains S.

2º. If $S = \varnothing$, then any subspace W contains S, and span$\{S\} = \{\mathbf{0}\}$ is contained in W. Now assume $S \neq \varnothing$ and assume $\boldsymbol{v}_i \in S \subset W$, $(1 \leq i \leq m)$. Then all multiples $a_i\boldsymbol{v}_i \in W$, $(1 \leq i \leq m)$ where $a_i \in \mathbb{R}$, and therefore the sum $(a_1\boldsymbol{v}_1 + \cdots + a_m\boldsymbol{v}_m) \in W$. That is, W contains all linear combinations of elements of S. Thus, span$\{S\} \subseteq W$, as claimed. □

Proposition 4. 16. *If W is a subspace of V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then span$\{W\} = W$.*

Proof. Since W is a subspace of V of the chemical equation (4. 2) over the field $\mathbb{R}$, W is closed under linear combinations. Hence span$\{W\} \subseteq W$. But $W \subseteq$ span$\{W\}$. Both inclusions yield span$\{W\} = W$. □

Proposition 4. 17. *If S is a subspace of V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then span$\{$span$\{S\}\}$ = span$\{S\}$.*

Proof. Since span$\{S\}$ is a subspace of V, the above Propositions 4. 16 implies that span$\{$span$\{S\}\}$ = span$\{S\}$. □

Proposition 4. 18. *If S and T are subsets of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, such that $S \subseteq T$, then span$\{S\} \subseteq$ span$\{T\}$.*

Proof. Assume $\boldsymbol{v} \in$ span$\{S\}$. Then

$$\boldsymbol{v} = a_1\boldsymbol{u}_1 + \cdots + a_r\boldsymbol{u}_r,$$

where $a_i \in \mathbb{R}$, $(1 \leq i \leq r)$ and $\boldsymbol{u}_i \in S$, $(1 \leq i \leq r)$. But $S \subseteq T$, therefore every $\boldsymbol{u}_i \in T$, $(1 \leq i \leq r)$. Thus $\boldsymbol{v} \in$ span$\{T\}$. Accordingly, span$\{S\} \subseteq$ span$\{T\}$. □

Proposition 4. 19. *The span$\{S\}$ is the intersection of all the subspaces of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$ which contains S.*

Proof. Let $\{W_i\}$ be the collection of all subspaces of a vector space V of the chemical equation (4. 2) containing S, and let $W = \cap W_i$. Since each W_i is a subspace of V, the set W is a subspace of V. Also, since each W_i contains S, the intersection W contains S. Hence span$\{S\} \subseteq W$. On the other hand, span$\{S\}$ is a subspace of V containing S. So span$\{S\} = W_k$ for some k. Then $W \subseteq W_k$ = span$\{S\}$. Both inclusions give span$\{S\} = W$. □

Proposition 4. 20. *If span$\{S\}$ = span$\{S \cup \{\mathbf{0}\}\}$, then one may delete the zero vector from any spanning set.*

Proof. By Proposition 4. 18, span$\{S\} \subseteq$ span$\{S \cup \{\mathbf{0}\}\}$. Assume $\boldsymbol{v} \in$ span$\{S \cup \{\mathbf{0}\}\}$, say

$$\boldsymbol{v} = a_1\boldsymbol{u}_1 + \cdots + a_n\boldsymbol{u}_n + b\cdot\mathbf{0}$$

where $a_i, b \in \mathbb{R}$, $(1 \leq i \leq n)$ and $\boldsymbol{u}_i \in S$, $(1 \leq i \leq n)$. Then

$$\boldsymbol{v} = a_1\boldsymbol{u}_1 + \cdots + a_n\boldsymbol{u}_n,$$

and so $\boldsymbol{v} \in$ span$\{S\}$. Thus span$\{S \cup \{\mathbf{0}\}\} \subseteq$ span$\{S\}$. Both inclusions give span$\{S\}$ = span$\{S \cup \{\mathbf{0}\}\}$. □

Proposition 4. 21. *If the vectors $\boldsymbol{v}_i \in V$, $(1 \leq i \leq n)$ span a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then for any vector $\boldsymbol{w} \in V$, the vectors $\boldsymbol{w}$, $\boldsymbol{v}_i$, $(1 \leq i \leq n)$ span V.*

Proof. Let $\boldsymbol{v} \in V$. Since the $\boldsymbol{v}_i$, $(1 \leq i \leq n)$ span V, there exist scalars a_i, $(1 \leq i \leq n)$ such that

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + \cdots + a_n\boldsymbol{v}_n + 0\boldsymbol{w}.$$

Thus $\boldsymbol{w}$, $\boldsymbol{v}_i$, $(1 \leq i \leq n)$ span V. □

Proposition 4. 22. *If $\boldsymbol{v}_i$, $(1 \leq i \leq n)$ span a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, and for $k > 1$, the vector $\boldsymbol{v}_k$ is a linear combination of the preceding vectors $\boldsymbol{v}_i$, $(1 \leq i \leq k - 1)$ then $\boldsymbol{v}_i$ without $\boldsymbol{v}_k$ span V, i. e.,*

$$\text{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_{k-1}, \boldsymbol{v}_{k+1}, \ldots, \boldsymbol{v}_n\} = V.$$

Proof. Let $\boldsymbol{v} \in V$. Since the $\boldsymbol{v}_i$, $(1 \leq i \leq n)$ span V, there exist scalars a_i, $(1 \leq i \leq n)$ such that

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + \cdots + a_n\boldsymbol{v}_n.$$

Since $\boldsymbol{v}_k$ is a linear combination of $\boldsymbol{v}_i$, $(1 \leq i \leq k - 1)$ there exist scalars b_i, $(1 \leq i \leq k - 1)$ such that

$$\boldsymbol{v}_k = b_1\boldsymbol{v}_1 + \cdots + a_{k-1}\boldsymbol{v}_{k-1}.$$

Thus

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + \cdots + a_k\boldsymbol{v}_k + \cdots + a_n\boldsymbol{v}_n$$

$$= a_1\boldsymbol{v}_1 + \cdots + a_k(b_1\boldsymbol{v}_1 + \cdots + b_{k-1}\boldsymbol{v}_{k-1}) + \cdots + a_n\boldsymbol{v}_n$$

$$= (a_1 + a_kb_1)\boldsymbol{v}_1 + \cdots + (a_{k-1} + a_kb_{k-1})\boldsymbol{v}_{k-1}$$

$$+ a_{k+1}\boldsymbol{v}_{k+1} + \cdots + a_n\boldsymbol{v}_n.$$

Therefore,

$$\text{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_{k-1}, \boldsymbol{v}_{k+1}, \ldots, \boldsymbol{v}_n\} = V. \ \square$$

Proposition 4. 23. *If W_i, $(1 \leq i \leq k)$ are subspaces of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, for which $W_1 \subset W_2 \subset \cdots \subset W_k$ and $W = W_1 \cup W_2 \cup \cdots \cup W_k$, then W is a subspace of V.*

Proof. The zero vector $\mathbf{0} \in W_1$, hence $\mathbf{0} \in W$. Assume $\boldsymbol{u}, \boldsymbol{v} \in W$. Then, there exist j_1 and j_2 such that $\boldsymbol{u} \in W_{j_1}$ and $\boldsymbol{v} \in W_{j_2}$. Let $j = \max(j_1, j_2)$. Then $W_{j_1} \subseteq W_j$ and $W_{j_2} \subseteq W$, and so $\boldsymbol{u}, \boldsymbol{v} \in W_j$. But W_j is a subspace. Therefore $(\boldsymbol{u} + \boldsymbol{v}) \in W_j$ and for any scalar s the multiple $s\boldsymbol{u} \in W_j$. Since $W_j \subseteq W$, we have $(\boldsymbol{u} + \boldsymbol{v})$, $s\boldsymbol{u} \in W$.

Thus W is a subspace of V. □

Proposition 4. 24. *If* W_i, $(1 \leq i \leq k)$ *are subspaces of a vector space* V *of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *and* S_i, $(1 \leq i \leq k)$ *span* W_i, $(1 \leq i \leq k)$ *then*

$$S = S_1 \cup S_2 \cup \cdots \cup S_k$$

spans W.

Proof. Let $\boldsymbol{v} \in W$. Then there exists j such that $\boldsymbol{v} \in W_j$. Then $\boldsymbol{v} \in \text{span}\{S_j\} \subseteq \text{span}\{S\}$. Therefore $W \subseteq \text{span}\{S\}$. But $S \subseteq W$ and W is a subspace. Hence $\text{span}\{S\} \subseteq W$. Both inclusions give $\text{span}\{S\} = W$, *i. e.*, S spans W. □

Theorem 4. 25. *Let* $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ *be a linearly independent set of vectors in a vector space* V *of the chemical equation* (4. 2) *over the field* $\mathbb{R}$, *then the following conditions*

1º $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ *is a linearly independent set,*

2º $\boldsymbol{v}$ *does not lie in* $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$, *are equivalent for a vector* $\boldsymbol{v}$ *in* V.

Proof. Assume 1º is true and assume, if possible, that $\boldsymbol{v}$ lies in $\text{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$, say,

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + \cdots + a_n\boldsymbol{v}_n.$$

Then

$$\boldsymbol{v} - a_1\boldsymbol{v}_1 + \cdots + a_n\boldsymbol{v}_n = \boldsymbol{0}$$

is a nontrivial linear combination, contrary to 1º. So 1º implies 2º. Conversely, assume that 2º holds and assume that

$$a\boldsymbol{v} + a_1\boldsymbol{v}_1 + \cdots + a_n\boldsymbol{v}_n = \boldsymbol{0}.$$

If $a \neq 0$, then

$$\boldsymbol{v} = (- a_1/a)\boldsymbol{v}_1 + \cdots + (- a_n/a)\boldsymbol{v}_n,$$

contrary to 2º. So $a = 0$ and

$$a_1\boldsymbol{v}_1 + \cdots + a_n\boldsymbol{v}_n = \boldsymbol{0}.$$

This implies that

$$a_1 = \cdots = a_n = 0,$$

because the set $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ is linearly independent. This proves that 2º implies 1º. □

Proposition 4. 26. *Let* $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ *be a linearly independent in a vector space* V *of the chemical equation* (4. 2) *over the field* $\mathbb{R}$, *then* $\{a_1\boldsymbol{v}_1, a_2\boldsymbol{v}_2, \ldots, a_n\boldsymbol{v}_n\}$ *is also linearly independent, such that the numbers* a_i, $(1 \leq i \leq n)$ *are all nonzero.*

Proof. Suppose a linear combination of the new set vanishes

$$s_1(a_1\boldsymbol{v}_1) + s_2(a_2\boldsymbol{v}_2) + \cdots + s_n(a_n\boldsymbol{v}_n) = \boldsymbol{0},$$

where s_i, $(1 \leq i \leq n)$ lie in $\mathbb{R}$.

Then

$$s_1a_1 = s_2a_2 = \cdots = s_na_n$$

by the linear independence of $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$. The fact that each $a_i \neq 0$ $(1 \leq i \leq n)$ now implies that

$$s_1 = s_2 = \cdots = s_n = 0.$$ □

Proposition 4. 27. *No linearly independent set of vectors of molecules can contain the zero vector.*

Proof. The set $\{\boldsymbol{0}, \boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ cannot be linearly independent, because

$$1\cdot\boldsymbol{0} + 0\boldsymbol{v}_1 + \cdots + 0\boldsymbol{v}_n = \boldsymbol{0},$$

is a non-trivial linear combination that vanishes. □

Theorem 4. 28. *A set* $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ *of vectors of molecules in a vector space* V *of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *is linearly dependent if and only if some* $\boldsymbol{v}_i$ *is a linear combination of the others.*

Proof. Assume that $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ is linearly dependent. Then, some nontrivial linear combination vanishes, *i. e.*,

$$a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n = \boldsymbol{0},$$

where some coefficient is not zero.

Suppose $a_1 \neq 0$. Then,

$$\boldsymbol{v}_1 = (- a_2/a_1)\boldsymbol{v}_2 + \cdots + (- a_n/a_1)\boldsymbol{v}_n$$

gives $\boldsymbol{v}_1$ as a linear combination of the others. In general, if $a_i \neq 0$, then a similar argument expresses $\boldsymbol{v}_i$ as linear combination of the others.

Conversely, suppose one of the vectors is a linear combination of the others, *i. e.*,

$$\boldsymbol{v}_1 = a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n.$$

Then, the nontrivial linear combination $1\boldsymbol{v}_1 - a_2\boldsymbol{v}_2 - \cdots - a_n\boldsymbol{v}_n$ equals zero, so the set $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ is not linearly independent, *i. e.*, it is linearly dependent. A similar argument works if *any* $\boldsymbol{v}_i$, $(1 \leq i \leq n)$ is linear combinations of the others. □

Theorem 4. 29. *Let* $V \neq 0$ *be a vector space of the chemical equation* (4. 2) *over the field* $\mathbb{R}$, *then*

1º *each set of linearly independent vectors is part of a basis of* V,

2º *each spanning set* V *contains a basis of* V,

3º V *has a basis and* $\dim V \leq n$.

Proof. 1º Really, if V is a vector space that is spanned by a finite number of vectors, we claim that *any* linearly independent subset $S = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k\}$ of V is contained in a basis of V. This is certainly true if $V = \text{span}\{S\}$ because then S is *itself* a basis of V. Otherwise, choose $\boldsymbol{v}_{k+1}$ outside $\text{span}\{S\}$. Then

$$S_1 = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k, \boldsymbol{v}_{k+1}\}$$

is linearly independent by Theorem 4. 25. If $V = \text{span}\{S_1\}$ then S_1 is the desired basis containing S. If not, choose $\boldsymbol{v}_{k+2}$ outside $\text{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k, \boldsymbol{v}_{k+1}\}$ so that

$$S_2 = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k, \boldsymbol{v}_{k+1}, \boldsymbol{v}_{k+2}\}$$

is linearly independent. Continue this process. Either a basis is reached at some stage or, if not, arbitrary large independent sets are found in V. But this later possibility cannot occur by the Theorem 4. 3 because V is spanned by a finite number of vectors.

2º Let

$$V = \text{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m\},$$

where (as $V \neq 0$) we may assume that each $\boldsymbol{v}_i \neq \boldsymbol{0}$. If $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m\}$ is linearly independent, it is itself a basis and we are finished. If not, then according to the Theorem 4. 28, one of these vectors lies in the span of the others. Relabeling if necessary, assume that $\boldsymbol{v}_1$ lies in $\text{span}\{\boldsymbol{v}_2, \ldots, \boldsymbol{v}_m\}$ so that

$$V = \text{span}\{\boldsymbol{v}_2, \ldots, \boldsymbol{v}_m\}.$$

Now repeat argument. If the set $\{\boldsymbol{v}_2, \ldots, \boldsymbol{v}_m\}$ is linearly independent, we are finished. If not, we have (after possible relabeling)

$$V = \text{span}\{\boldsymbol{v}_3, \ldots, \boldsymbol{v}_m\}.$$

Continue this process. If a basis is encountered at some stage, we are finished. If not, we ultimately reach

$$V = \text{span}\{\boldsymbol{v}_m\}.$$

But then $\{\boldsymbol{v}_m\}$ is a basis because $\boldsymbol{v}_m \neq \boldsymbol{0}$ ($V \neq 0$).

3º V has a spanning set of n vectors, one of which is nonzero because $V \neq 0$. Hence 3º follows from 2º. □

Corollary 4. 30. *A nonzero vector space V of the chemical equation* (4. 2) *over the field* ℝ *is finite dimensional if only if it can be spanned by finitely many vectors.* □

Theorem 4. 31. *Let V be a vector space of the chemical equation* (4. 2) *over the field* ℝ *and* $\dim V = n > 0$, *then*

1º *no set of more than n vectors in V can be linearly independent,*

2º *no set of fewer than n vectors can span V.*

Proof. V can be spanned by n vectors (any basis) so 1º restates the Theorem 4. 3. But the n basis vectors are also linearly independent, so no spanning set can have fewer than n vectors, again by Theorem 4. 3. This gives 2º. □

Theorem 4. 32. *Let V be a vector space of the chemical equation* (4. 2) *over the field* ℝ *and* $\dim V = n > 0$, *then*

1º *any set of n linearly independent vectors in V is a basis* (*that is, it necessarily spans V*),

2º *any spanning set of n nonzero vectors in V is a basis* (*that is, it necessarily linearly independent*).

Proof. 1º If the n independent vectors do not span V, they are part of a basis of more than n vectors by property 1º of the Theorem 4. 29. This contradicts Theorem 4. 31.

2º If the n vectors in a spanning set are not linearly independent, they contain a basis of fewer than n vectors by property 2º of Theorem 4. 29, contradicting Theorem 4. 31. □

Theorem 4. 33. *Let V be a vector space of dimension n of the chemical equation* (4. 2) *over the field* ℝ *and let U and W denote subspaces of V, then*

1º *U is finite dimensional and* $\dim U \leq n$,

2º *any basis of U is part of a basis for V,*

3º *if* $U \subseteq W$ *and* $\dim U = \dim W$, *then* $U = W$.

Proof. 1º If $U = 0$, $\dim U = 0$ by Definition 3. 11. So assume $U \neq 0$ and choose $\boldsymbol{u}_1 \neq \boldsymbol{0}$ in U. If $U = \text{span}\{\boldsymbol{u}_1\}$, then $\dim U = 1$. If $U \neq \text{span}\{\boldsymbol{u}_1\}$, choose $\boldsymbol{u}_2$ in U outside $\text{span}\{\boldsymbol{u}_1\}$. Then $\{\boldsymbol{u}_1, \boldsymbol{u}_2\}$, is linearly independent by Theorem 4. 25. If $U = \text{span}\{\boldsymbol{u}_1, \boldsymbol{u}_2\}$, then $\dim U = 2$. If not, repeat the process to find $\boldsymbol{u}_3$ in U such that $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$ is linearly independent. Continue in this way. The process must terminate because the space V (having dimension n) cannot contain more than n independent vectors. Therefore U has a basis of at most n vectors, proving 1º.

2º This follows from 1º and Theorem 4. 29.

3º Let $\dim U = \dim W = m$. Then any basis $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_m\}$ of U is an independent set of m vectors in W and so is a basis of W by Theorem 4. 32. In particular, $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_m\}$ spans W so, because it also spans U, $W = \text{span}\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_m\} = U$. This proves 3º. □

Proposition 4. 34. *If U and W are subspaces of a vector space V of the chemical equation* (4. 2) *over the field* ℝ, *then* $U + W$ *is a subspace of V.*

Proof. Since U and W are subspaces, $\boldsymbol{0} \in U$ and $\boldsymbol{0} \in W$. Hence $\boldsymbol{0} = \boldsymbol{0} + \boldsymbol{0} \in U + W$. Assume $\boldsymbol{v}, \boldsymbol{v}' \in U + W$. Then there exist $\boldsymbol{u}, \boldsymbol{u}' \in U$ and $\boldsymbol{v}, \boldsymbol{v}' \in W$ such that $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w}$ and $\boldsymbol{v}' = \boldsymbol{u}' + \boldsymbol{w}'$.

Since U and W are subspaces, $\boldsymbol{u} + \boldsymbol{u}' \in U$ and $\boldsymbol{w} + \boldsymbol{w}' \in W$ and for any scalar k, $k\boldsymbol{u} \in U$ and $k\boldsymbol{w} \in W$.

Accordingly,

$$\boldsymbol{v} + \boldsymbol{v}' = (\boldsymbol{u} + \boldsymbol{w}) + (\boldsymbol{u}' + \boldsymbol{w}')$$
$$= (\boldsymbol{u} + \boldsymbol{u}') + (\boldsymbol{w} + \boldsymbol{w}') \in U + W$$

and for any scalar k,

$$k\boldsymbol{v} = k(\boldsymbol{u} + \boldsymbol{w}) = k\boldsymbol{u} + k\boldsymbol{w} \in U + W.$$

Thus $U + W$ is a subspace of V. □

Proposition 4. 35. *If U and W are subspaces of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then U and W are contained in $U + W$.*

Proof. Let $\boldsymbol{u} \in U$. By hypothesis W is a subspace of V and so $\boldsymbol{0} \in W$. Hence $\boldsymbol{u} = \boldsymbol{u} + \boldsymbol{0} \in U + W$. Accordingly, U is contained in $U + W$. Similarly, W is contained in $U + W$. □

Proposition 4. 36. *If U and W are subspaces of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then $U + W$ is the smallest subspace of V containing U and V, i. e., $U + W = span\{U, W\}$.*

Proof. Since $U + W$ is a subspace of V containing both U and W, it must also contain the linear span of U and W, *i. e.*, $\text{span}\{U, W\} \subseteq U + W$.

On the other hand, if $\boldsymbol{v} \in U + W$ then

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w} = 1\boldsymbol{u} + 1\boldsymbol{w},$$

where $\boldsymbol{u} \in U$ and $\boldsymbol{w} \in W$. Hence, $\boldsymbol{v}$ is a linear combination of elements in $U \cup W$ and so belongs to $\text{span}\{U, W\}$.

Therefore

$$U + W \subseteq \text{span}\{U, W\}.$$

Both inclusions give us the required result.□

Proposition 4. 37. *If W is a subspace of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then $W + W = W$.*

Proof. Since W is a subspace of V, we have that W is closed under vector addition.

Therefore

$$W + W \subseteq W.$$

By Proposition 4. 35,

$$W \subseteq W + W.$$

Thus,

$$W + W = W.$$ □

Proposition 4. 38. *If U and W are subspaces of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, such that $U = span\{S\}$ and $W = span\{T\}$, then $U + W = span\{S \cup T\}$.*

Proof. Since

$$S \subseteq U \subseteq U + W$$

and

$$T \subseteq W \subseteq U + W,$$

we have

$$S \cup T \subseteq U + W.$$

Hence

$$\text{span}\{S \cup T\} \subseteq U + W.$$

Now assume $\boldsymbol{v} \in U + W$. Then $\boldsymbol{v} = \boldsymbol{u} + W$, where $\boldsymbol{u} \in U$ and $\boldsymbol{w} \in W$.

Since

$$U = \text{span}\{S\} \text{ and } W = \text{span}\{T\},$$

$$\boldsymbol{u} = a_1\boldsymbol{u}_1 + \cdots + a_r\boldsymbol{u}_r \text{ and } \boldsymbol{w} = b_1\boldsymbol{w}_1 + \cdots + b_s\boldsymbol{w}_s,$$

where $a_i, b_j \in \mathbb{R}$, $\boldsymbol{u}_j \in S$, and $\boldsymbol{w}_i \in T$.

Then

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w} = a_1\boldsymbol{u}_1 + \cdots + a_r\boldsymbol{u}_r + b_1\boldsymbol{w}_1 + \cdots + b_s\boldsymbol{w}_s.$$

Thus,

$$U + W \subseteq \text{span}\{S \cup T\}.$$

Both inclusions yield

$$U + W = span\{S \cup T\}.$$ □

Proposition 4. 39. *If U and W are subspaces of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then $V = U + W$ if every $\boldsymbol{v} \in V$ can be written in the form $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w}$, where $\boldsymbol{u} \in U$ and $\boldsymbol{w} \in W$.*

Proof. Assume, for any $\boldsymbol{v} \in V$, we have $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w}$ where $\boldsymbol{u} \in U$ and $\boldsymbol{w} \in W$. Then $\boldsymbol{v} \in U + W$ and so $V \subseteq U + W$. Since U and V are subspaces of V, we have $U + W \subseteq V$. Both inclusions imply $V = U + W$. □

Theorem 4. 40. *The vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, is the direct sum of its subspaces U and W, if only if*

$$V = U + W \text{ and } U \cap W = \{\boldsymbol{0}\}.$$

Proof. Assume $V = U \oplus W$. Then any $\boldsymbol{v} \in V$ can be uniquely written in the form $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w}$ where $\boldsymbol{u} \in U$ and $\boldsymbol{w} \in W$. Thus, in particular, $V = U + W$. Now assume $\boldsymbol{v} \in U \cap W$. Then $\boldsymbol{v} = \boldsymbol{v} + \boldsymbol{0}$, where $\boldsymbol{v} \in U$, $\boldsymbol{0} \in W$ and $\boldsymbol{v} = \boldsymbol{0} + \boldsymbol{v}$, where $\boldsymbol{0} \in U$, $\boldsymbol{v} \in W$. Since such a sum for $\boldsymbol{v}$ must be unique, $\boldsymbol{v} = \boldsymbol{0}$. Thus, $U \cap W = \{\boldsymbol{0}\}$.

On the other hand, assume $V = U + W$ and $U \cap W = \{\boldsymbol{0}\}$. Let $\boldsymbol{v} \in V$. Since $V = U + W$, there exist $\boldsymbol{u} \in U$ and $\boldsymbol{w} \in W$, such that $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w}$. We need to show that such a sum is unique. Assume also that $\boldsymbol{v} = \boldsymbol{u}' + \boldsymbol{w}'$ where $\boldsymbol{u}' \in U$ and $\boldsymbol{w}' \in W$. Then $\boldsymbol{u} + \boldsymbol{w} = \boldsymbol{u}' + \boldsymbol{w}'$ and so $\boldsymbol{u} - \boldsymbol{u}' = \boldsymbol{w}' - \boldsymbol{w}$. But, $\boldsymbol{u} - \boldsymbol{u}' \in U$ and $\boldsymbol{w}' - \boldsymbol{w} \in W$. Hence by $U \cap W = \{\boldsymbol{0}\}$, $\boldsymbol{u} - \boldsymbol{u}' = \boldsymbol{0}$, $\boldsymbol{w}' - \boldsymbol{w} = \boldsymbol{0}$ and so $\boldsymbol{u} = \boldsymbol{u}'$, $\boldsymbol{w}' = \boldsymbol{w}$. Thus such a sum for $\boldsymbol{v} \in V$ is unique and $V = U \oplus W$. □

Proposition 4. 41. *Let W_1, W_2, …, W_r be subspaces of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, such that*

$$V = W_1 + W_2 + \cdots + W_r$$

and $\boldsymbol{0} \in V$ and let be written uniquely as a sum

$$\boldsymbol{0} = \boldsymbol{w}_1 + \boldsymbol{w}_2 + \cdots + \boldsymbol{w}_r,$$

where $\boldsymbol{w}_i \in W_i$, $(1 \le i \le r)$, then

$$V = W_1 \oplus W_2 \oplus \cdots \oplus W_r,$$

i. e., that the sum is direct.

Proof. Since

$$\mathbf{0} = \mathbf{0}_1 + \mathbf{0}_2 + \cdots + \mathbf{0}_r,$$

where $\mathbf{0}_i \in W_i$, $(1 \le i \le r)$, this is the unique sum for $\mathbf{0} \in V$. Let $\boldsymbol{v} \in V$ and assume

$$\boldsymbol{v} = \boldsymbol{u}_1 + \boldsymbol{u}_2 + \cdots + \boldsymbol{u}_r$$

and

$$\boldsymbol{v} = \boldsymbol{w}_1 + \boldsymbol{w}_2 + \cdots + \boldsymbol{w}_r$$

where $\boldsymbol{u}_i, \boldsymbol{w}_i \in W_i$, $(1 \le i \le r)$. Then

$$\mathbf{0} = \boldsymbol{v} - \boldsymbol{v}$$
$$= (\boldsymbol{u}_1 - \boldsymbol{w}_1) + (\boldsymbol{u}_2 - \boldsymbol{w}_2) + \cdots + (\boldsymbol{u}_r - \boldsymbol{w}_r),$$

where $(\boldsymbol{u}_i - \boldsymbol{w}_i) \in W_i$, $(1 \le i \le r)$. Since such a sum for $\mathbf{0}$ is unique, $\boldsymbol{u}_i - \boldsymbol{v}_i = \mathbf{0}$, $(1 \le i \le r)$ and hence $\boldsymbol{u}_i = \boldsymbol{v}_i$, $(1 \le i \le r)$. Thus, such a sum for $\boldsymbol{v}$ is also unique, and so

$$V = W_1 \oplus W_2 \oplus \cdots \oplus W_r. \qquad \square$$

Theorem 4. 42. *Let Π_i, $(1 \le i \le m)$ be the hyperplanes of the chemical equation* (4. 1), *which is reduced to the linear system* (4. 8), *in an m-dimensional real space $\mathbb{R}^m$ and let W_i, $(1 \le i \le m)$ be directions of these hyperplanes, then*

$$\dim(\sum_{1\le i\le m} \Pi_i) = \sum_{1\le i\le m} \dim\Pi_i \qquad (4.\ 14)$$
$$- \sum_{1\le i<j\le m} \dim(\Pi_i \cap \Pi_j)$$
$$+ \sum_{1\le i<j<k\le n} \dim(\Pi_i \cap \Pi_j \cap \Pi_k) - \cdots$$
$$+ (-1)^{n-1}\dim(\bigcap_{1\le i\le m} \Pi_i),\ (\Pi_i \cap \Pi_j \ne \varnothing,\ i \ne j)$$

and

$$\dim(\sum_{1\le i\le m} \Pi_i) = \sum_{1\le i\le m} \dim\Pi_i \qquad (4.\ 15)$$
$$- \sum_{1\le i<j\le m} \dim(W_i \cap W_j)$$
$$+ \sum_{1\le i<j<k\le m} \dim(W_i \cap W_j \cap W_k) - \cdots$$
$$+ (-1)^{n-1}\dim(\bigcap_{1\le i\le m} W_i) + 1,\ (\Pi_i \cap \Pi_j = \varnothing,\ i \ne j).$$

Proof. First we shall proof the identity (4. 14). We apply induction. For $m = 2$, we obtain the Grassmann's formula

$$\dim(\Pi_1 + \Pi_2) = \dim\Pi_1 + \dim\Pi_2 \quad (4.16)$$
$$- \dim(\Pi_1 \cap \Pi_2),\ (\Pi_1 \cap \Pi_2 \ne \varnothing).$$

The truthfulness of the formula (4. 16) follows from the formula

$$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 \ (4.17)$$
$$- \dim(W_1 \cap W_2),\ (W_1 \cap W_2 \ne \varnothing),$$

that holds for vector spaces, because

$$\dim(W_1 \cap W_2) = \dim(\Pi_1 \cap \Pi_2), \quad (4.18)$$

where W_1 and W_2 are subspaces of the vector space V.

Now we shall prove the identity (4. 17). If W_1 and W_2, of dimensions $r \le m$ and $s \le m$, respectively, are subspaces of a vector space V of dimension n and $W_1 \cap W_2$ and $W_1 + W_2$ are of dimensions p and t, respectively, then $t = r + s - p$.

Take $A = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_p\}$ as a basis of $W_1 \cap W_2$ and take $B = A \cup \{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_{r-p}\}$ as a basis of W_1 and $C = A \cup \{\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_{s-p}\}$ as a basis of W_2. Then, any vector of $W_1 + W_2$ can be expressed as a linear combination of the vectors of

$D = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_p, \boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_{r-p}, \boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_{s-p}\}$.

To show that D is a linearly independent set and, hence, is a basis of $W_1 + W_2$, consider

$$a_1\boldsymbol{x}_1 + a_2\boldsymbol{x}_2 + \cdots + a_p\boldsymbol{x}_p \qquad (4.19)$$
$$+ b_1\boldsymbol{y}_1 + b_2\boldsymbol{y}_2 + \cdots + b_{r-p}\boldsymbol{y}_{r-p}$$
$$+ c_1\boldsymbol{z}_1 + c_2\boldsymbol{z}_2 + \cdots + c_{s-p}\boldsymbol{z}_{s-p} = \mathbf{0},$$

where $a_i, b_j, c_k \in \mathbb{R}$.

Set

$$\boldsymbol{\pi} = c_1\boldsymbol{z}_1 + c_2\boldsymbol{z}_2 + \cdots + c_{s-p}\boldsymbol{z}_{s-p}.$$

Now $\boldsymbol{\pi} \in W_2$ and by (4. 19) $\boldsymbol{\pi} \in W_1$. Thus, $\boldsymbol{\pi} \in W_1 \cap W_2$ and is a linear combination of the vectors of A, say

$$\boldsymbol{\pi} = d_1\boldsymbol{x}_1 + d_2\boldsymbol{x}_2 + \cdots + d_p\boldsymbol{x}_p,\ d_i \in \mathbb{R}.$$

Then

$$c_1\boldsymbol{z}_1 + c_2\boldsymbol{z}_2 + \cdots + c_{s-p}\boldsymbol{z}_{s-p} - d_1\boldsymbol{x}_1 - d_2\boldsymbol{x}_2 - \cdots - d_p\boldsymbol{x}_p = \mathbf{0}$$

and, since C is a basis of W_2, each $c_i = 0$ and each $d_i = 0$. With each $c_i = 0$, (4. 19) becomes

$$a_1\boldsymbol{x}_1 + a_2\boldsymbol{x}_2 + \cdots + a_p\boldsymbol{x}_p \qquad (4.20)$$
$$+ b_1\boldsymbol{y}_1 + b_2\boldsymbol{y}_2 + \cdots + b_{r-p}\boldsymbol{y}_{r-p} = \mathbf{0}.$$

Since B is a basis of W_1, each $a_i = 0$ and each $b_i = 0$ in (4. 20). Then D is a linearly independent set and, hence, is a basis of $W_1 + W_2$ of dimension $t = r + s - p$.

We assume that (4. 14) is true for m and we shall use this assumption to deduce that (4. 14) is true for $m + 1$.

Next, we have

$$\dim(\Pi_1 + \Pi_2 + \cdots + \Pi_m + \Pi_{m+1})$$
$$= \dim[(\Pi_1 + \Pi_2 + \cdots + \Pi_m) + \Pi_{m+1}]$$
$$= \dim(\Pi_1 + \Pi_2 + \cdots + \Pi_m) + \dim(\Pi_{m+1})$$
$$- \dim[(\Pi_1 + \Pi_2 + \cdots + \Pi_m) \cap \Pi_{m+1}]$$
$$= \dim\Pi_1 + \dim\Pi_2 + \cdots + \dim\Pi_m$$
$$- \dim(\Pi_1 \cap \Pi_2) - \dim(\Pi_1 \cap \Pi_3) - \cdots$$
$$- \dim(\Pi_{m-1} \cap \Pi_m) + \cdots$$
$$+ (-1)^{m-1}\dim(\Pi_1 \cap \Pi_2 \cap \cdots \cap \Pi_m)$$

$$+ \dim(\Pi_{m+1}) - \dim(\Pi_1 \cap \Pi_{m+1} + \Pi_2 \cap \Pi_{m+1} + \cdots + \Pi_m \cap \Pi_{m+1})$$
$$= \dim\Pi_1 + \dim\Pi_2 + \cdots + \Pi_{m+1}$$
$$- \dim(\Pi_1 \cap \Pi_2) - \dim(\Pi_1 \cap \Pi_3) - \cdots$$
$$- \dim(\Pi_{m-1} \cap \Pi_m) + \cdots$$
$$+ (-1)^{m-1}\dim(\Pi_1 \cap \Pi_2 \cap \cdots \cap \Pi_m)$$
$$- \{\dim(\Pi_1 \cap \Pi_{m+1}) + \dim(\Pi_2 \cap \Pi_{m+1}) + \cdots + \dim(\Pi_m \cap \Pi_{m+1})$$
$$- \dim[(\Pi_1 \cap \Pi_{m+1}) \cap (\Pi_2 \cap \Pi_{m+1})]$$
$$- \dim[(\Pi_2 \cap \Pi_{m+1}) \cap (\Pi_3 \cap \Pi_{m+1})] - \cdots$$
$$- \dim[(\Pi_{m-1} \cap \Pi_{m+1}) \cap (\Pi_m \cap \Pi_{m+1})]$$
$$+ \cdots + (-1)^{m-1}\dim[(\Pi_1 \cap \Pi_{m+1}) \cap (\Pi_2 \cap \Pi_{m+1}) \cap \cdots \cap (\Pi_m \cap \Pi_{m+1})]\}$$
$$= \dim\Pi_1 + \dim\Pi_2 + \cdots + \dim\Pi_{m+1}$$
$$- \dim(\Pi_1 \cap \Pi_2) - \dim(\Pi_1 \cap \Pi_3) - \cdots$$
$$- \dim(\Pi_{m-1} \cap \Pi_m) + \cdots$$
$$+ (-1)^{m-1}\dim(\Pi_1 \cap \Pi_2 \cap \cdots \cap \Pi_m)$$
$$- \dim(\Pi_1 \cap \Pi_{m+1}) - \dim(\Pi_2 \cap \Pi_{m+1}) - \cdots$$
$$- \dim(\Pi_m \cap \Pi_{m+1}) + \dim(\Pi_1 \cap \Pi_2 \cap \Pi_{m+1})$$
$$+ \dim(\Pi_2 \cap \Pi_3 \cap \Pi_{m+1}) + \cdots$$
$$+ \dim(\Pi_{m-1} \cap \Pi_m \cap \Pi_{m+1}) - \cdots$$
$$+ (-1)^m\dim(\Pi_1 \cap \Pi_2 \cap \cdots \cap \Pi_{m+1})$$
$$= \dim\Pi_1 + \dim\Pi_2 + \cdots + \dim\Pi_{m+1}$$
$$- \dim(\Pi_1 \cap \Pi_2) - \dim(\Pi_1 \cap \Pi_3) - \cdots$$
$$- \dim(\Pi_m \cap \Pi_{m+1}) + \dim(\Pi_1 \cap \Pi_2 \cap \Pi_{m+1})$$
$$+ \dim(\Pi_2 \cap \Pi_3 \cap \Pi_{m+1}) + \cdots$$
$$+ \dim(\Pi_{m-1} \cap \Pi_m \cap \Pi_{m+1}) + \cdots$$
$$+ (-1)^m\dim(\Pi_1 \cap \Pi_2 \cap \cdots \cap \Pi_{m+1})$$
$$= \sum_{1 \le i \le m+1} \dim\Pi_i - \sum_{1 \le i < j \le m+1} \dim(\Pi_i \cap \Pi_j)$$
$$+ \sum_{1 \le i < j < k \le m+1} \dim(\Pi_i \cap \Pi_j \cap \Pi_k)$$
$$- \cdots + (-1)^m\dim(\bigcap_{1 \le i \le m+1} \Pi_i), \ (\Pi_i \cap \Pi_j \neq \varnothing, \ i \neq j).$$

By this the identity (4. 14) is proved.

Now, we shall continue with induction in order to prove the identity (4.15). For $m = 2$, the identity (4.15) reduces to this form

$$\dim(\Pi_1 + \Pi_2) = \dim\Pi_1 + \dim\Pi_2 - \dim(W_1 \cap W_2) + 1, \ (\Pi_1 \cap \Pi_2 = \varnothing). \quad (4.21)$$

Let $\Pi_1 \cap \Pi_2 = \varnothing$. Let $\dim\Pi_1 = r$ and $\dim\Pi_2 = s$. Let the point $X \in \Pi_1$ and the point $Y \in \Pi_2$ and let the basis of the subspace W_2 is the set $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_s\}$. Obviously, the vector $\boldsymbol{YX} \notin W_2$, because conversely $X \in \Pi_1 \cap \Pi_2$ what is a contradiction.

Therefore, the set

$$S = \{\boldsymbol{YX}, \boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_s\}$$

is a basis of the space $W_3 = [\boldsymbol{XY}] + W_2$ and $\dim W_3 = s + 1$. Hyperplane of the point X, which determine the subspace W_3 is given by $X + \Pi_2$, such that $\dim(X + \Pi_2) = s + 1$.

Since,

$$\Pi_1 \cap (X + \Pi_2) \neq \varnothing,$$

then from (4.16) follows

$$\dim(\Pi_1 + \Pi_2) = \dim[\Pi_1 + (X + \Pi_2)] \quad (4.22)$$
$$= r + (s + 1) - \dim[\Pi_1 + (X + \Pi_2)].$$

Next, since (4.18) we obtain

$$\dim[\Pi_1 + (X + \Pi_2)] = \dim(W_1 \cap W_3).$$

Let $Y \in \Pi_2$. Prove that $\boldsymbol{XY} \notin W_1 + W_2$. Assume the contrary, *i. e.*, $\boldsymbol{XY} \in W_1 + W_2$, then there exist two vectors $\boldsymbol{x} \in W_1$ and $\boldsymbol{y} \in W_2$ such that $\boldsymbol{XY} = \boldsymbol{x} + \boldsymbol{y}$. Then, there exists a point $Z \in \Pi_1$ such that $\boldsymbol{XZ} = \boldsymbol{x}$ and according to the definition of affine space, one obtain

$$\boldsymbol{XY} = \boldsymbol{XZ} + \boldsymbol{ZY} = \boldsymbol{x} + \boldsymbol{ZY},$$

i. e., $\boldsymbol{ZY} = \boldsymbol{y} \in W_2$. Therefore $Z \in \Pi_2$, respectively $Z \in \Pi_1 + \Pi_2$, that is a contradiction with the supposition $\Pi_1 \cap \Pi_2 = \varnothing$.

By this we proved that $\boldsymbol{XY} \notin W_1 + W_2$. Since $W_3 = [\boldsymbol{XY}, \boldsymbol{v}_1, \ldots, \boldsymbol{v}_s]$, then holds

$$W_1 \cap W_3 = W_1 \cap W_2,$$

and from (4.22) follows

$$\dim(\Pi_1 + \Pi_2) = r + s + 1 - \dim(W_1 \cap W_2),$$

i. e., holds (4. 15).

Since the identity (4.15) holds for $m = 2$, we suppose that it is true for arbitrary m. By virtue of inductive hypothesis we shall prove its truthfulness for $m + 1$, *i. e.*,

$$\dim(\Pi_1 + \Pi_2 + \cdots + \Pi_m + \Pi_{m+1})$$
$$= \dim[(\Pi_1 + \Pi_2 + \cdots + \Pi_m) + \Pi_{m+1}]$$
$$= \dim(\Pi_1 + \Pi_2 + \cdots + \Pi_m) + \dim\Pi_{m+1}$$
$$- \dim[(W_1 + W_2 + \cdots + W_m) \cap W_{m+1}] + 1$$
$$= \dim(\Pi_1 + \Pi_2 + \cdots + \Pi_m) + \dim\Pi_{m+1}$$
$$- [\dim(W_1 \cap W_{m+1} + W_2 \cap W_{m+1} + \cdots + W_m \cap W_{m+1})] + 1$$
$$= \dim\Pi_1 + \dim\Pi_2 + \cdots + \dim\Pi_m + \dim\Pi_{m+1} - \dim(W_1 \cap W_2)$$
$$- \dim(W_1 \cap W_3) - \cdots - \dim(W_{m-1} \cap W_m) + \cdots$$
$$+ (-1)^{m-1}\dim(W_1 \cap W_2 \cap \cdots \cap W_m) + 1$$
$$- \{\dim(W_1 \cap W_{m+1}) + \dim(W_2 \cap W_{m+1}) + \cdots + \dim(W_m \cap W_{m+1})$$
$$- \dim[(W_1 \cap W_{m+1}) \cap (W_2 \cap W_{m+1})]$$
$$- \dim[(W_2 \cap W_{m+1}) \cap (W_3 \cap W_{m+1})] - \cdots$$
$$- \dim[(W_{m-1} \cap W_{m+1}) \cap (W_m \cap W_{m+1})]$$
$$+ \cdots + (-1)^{m-1}\dim[(W_1 \cap W_{m+1}) \cap (W_2 \cap W_{m+1})$$

$$\cap \cdots \cap (W_m \cap W_{m+1})] + 1\} + 1$$
$$= \dim\Pi_1 + \dim\Pi_2 + \cdots + \dim\Pi_{m+1}$$
$$- \dim(W_1 \cap W_2)$$
$$- \dim(W_1 \cap W_3) - \cdots - \dim(W_{m-1} \cap W_m)$$
$$+ \cdots + (-1)^{m-1}\dim(W_1 \cap W_2 \cap \cdots \cap W_m)$$
$$+ 1 - \dim(W_1 \cap W_{m+1}) - \dim(W_2 \cap W_{m+1}) - \cdots$$
$$- \dim(W_m \cap W_{m+1}) + \dim(W_1 \cap W_2 \cap W_{m+1})$$
$$+ \dim(W_2 \cap W_3 \cap W_{m+1}) + \cdots$$
$$+ \dim(W_{m-1} \cap W_m \cap W_{m+1}) + \cdots$$
$$- (-1)^{m-1}\dim(W_1 \cap W_2 \cap \cdots \cap W_m \cap W_{m+1})$$
$$- 1 + 1 = \dim\Pi_1 + \dim\Pi_2 + \cdots + \dim\Pi_{m+1}$$
$$- \dim(W_1 \cap W_2) - \dim(W_1 \cap W_3)$$
$$- \cdots - \dim(W_m \cap W_{m+1}) + \dim(W_1 \cap W_2 \cap W_{m+1})$$
$$+ \dim(W_2 \cap W_3 \cap W_{m+1}) + \cdots$$
$$+ \dim(W_{m-1} \cap W_m \cap W_{m+1}) + \cdots$$
$$+ (-1)^m\dim(\Pi_1 \cap \Pi_2 \cap \cdots \cap \Pi_{m+1}) + 1$$
$$= \sum_{1 \le i \le m+1} \dim\Pi_i - \sum_{1 \le i < j \le m+1} \dim(W_i \cap W_j)$$
$$+ \sum_{1 \le i < j < k \le m+1} \dim(W_i \cap W_j \cap W_k) - \cdots$$
$$+ (-1)^{m-1}\dim(\bigcap_{1 \le i \le m+1} W_i) + 1, \ (\Pi_i \cap \Pi_j = \varnothing, i \neq j).$$

The proof of the theorem is completed. □

Remark 4. 43. *This theorem generalizes well-known Grassmann's formulas given in* [22].

As a natural consequence of proved theorem appears the following corollary.

Corollary 4. 44. *Let* $\Pi_i^{k_i}$, $(1 \le i \le m)$ *be* k_i-*dimensional hyperplanes of the chemical equation* (4. 1), *which is reduced to the linear system* (4. 8), *in an m-dimensional real space* $\mathbb{R}^m$. *Then*

$$\dim(\sum_{1 \le i \le m} \Pi_i^{k_i}) \le \sum_{1 \le i \le m} k_i + 1. \qquad (4.23)$$

The sign equality in (4.23) *holds if only if the hyperplanes* Π_i^{ki} *and* Π_j^{kj}, $(i \neq j)$, *are disjoint and the sum of subspaces* $\Pi_i^{ki} \preccurlyeq V^m$, $(1 \le i \le m)$ *is direct.* □

Proposition 4. 45. *Let*

$$V = U_1 \oplus U_2 \oplus \cdots \oplus U_r$$

is a vector space of the chemical equation (4. 2) *over the field* $\mathbb{R}$ *and* B_i, $(1 \le i \le r)$ *is a basis of* U_i, $(1 \le i \le r)$, *then* $B = \cup B_i$, $(1 \le i \le r)$ *is a basis of V.*

Proof. B is linearly independent since each B_i is linearly independent. Assume $\boldsymbol{v} \in V$. Then

$$\boldsymbol{v} = \boldsymbol{u}_1 + \boldsymbol{u}_2 + \cdots + \boldsymbol{u}_r,$$

where $\boldsymbol{u}_i \in U_i$, $(1 \le i \le r)$. Then $\boldsymbol{u}_i$, $(1 \le i \le r)$ is a linear combination of the vectors in B_i, $(1 \le i \le r)$. Therefore $\boldsymbol{v}$ is a linear combination of the vectors in B.

Thus B spans V. Since B is linearly independent and spans V, B is a basis of V. □

Proposition 4. 46. *Let*

$$V = U_1 \oplus U_2 \oplus \cdots \oplus U_r$$

is a vector space of the chemical equation (4. 2) *over the field* $\mathbb{R}$, *where* $dimU_i = n_i$, *then*

$$dimV = dimU_1 + dimU_2 + \cdots + dimU_r.$$

Proof. Let B_i be a basis of U_i, $(1 \le i \le r)$. Hence B_i, $(1 \le i \le r)$ has n_i, $(1 \le i \le r)$ elements. Thus $B = \cup B_i$, $(1 \le i \le r)$ has $n_1 + n_2 + \cdots + n_r$ elements. By Proposition 4. 45, B is a basis of V.

Hence

$$\dim V = n_1 + n_2 + \cdots + n_r$$
$$= dimU_1 + dimU_2 + \cdots + dimU_r. \quad □$$

Proposition 4. 47. *If* $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_r, \boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_s\}$ *is a linearly independent subset of a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$, *then*

$$span\{\boldsymbol{u}_i\} \cap span\{\boldsymbol{w}_j\} = \{\boldsymbol{0}\}.$$

Proof. Assume $\boldsymbol{v} \in \text{span}\{\boldsymbol{u}_i\} \cap \text{span}\{\boldsymbol{w}_j\}$. Then there exist scalars a_i, $(1 \le i \le r)$ and b_j, $(1 \le j \le s)$ such that

$$\boldsymbol{v} = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_r\boldsymbol{u}_r$$
$$= b_1\boldsymbol{w}_1 + b_2\boldsymbol{w}_2 + \cdots + b_s\boldsymbol{w}_s.$$

Hence

$$a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_r\boldsymbol{u}_r$$
$$- b_1\boldsymbol{w}_1 - b_2\boldsymbol{w}_2 - \cdots - b_s\boldsymbol{w}_s = \boldsymbol{0}.$$

But $\{\boldsymbol{u}_i, \boldsymbol{w}_j\}$ is linearly independent. Hence each $a_i = 0$, $(1 \le i \le r)$ and each $b_j = 0$, $(1 \le j \le r)$. Thus $\boldsymbol{v} = \boldsymbol{0}$. □

Proposition 4. 48. *Let U be a subspace of a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$, *then exists a subspace W of V, such that* $V = U \oplus W$.

Proof. Let $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_r\}$ be a basis of U. Since $\{\boldsymbol{u}_i\}$, $(1 \le i \le r)$ is linearly independent, it can be extended to a basis of V, *i. e.*, $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_r, \boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_s\}$. Let W be the space generated by $\{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_s\}$. Since $\{\boldsymbol{u}_i, \boldsymbol{w}_j\}$, $(1 \le i \le r, 1 \le j \le s)$ spans V, we have $V = U + W$. On the other hand, by Proposition 4. 47, $U \cap W = \{\boldsymbol{0}\}$. Accordingly, $V = U \oplus W$. □

Proposition 4. 49. *Let B is a linearly independent subset of a vector space V of the* *chemical equation* (4. 2) *over the field* $\mathbb{R}$ *and let* $[B_1, B_2, \ldots, B_r]$ *be a partition of B, then*

$$span\{B\} = span\{B_1\} \oplus span\{B_2\} \oplus \cdots \oplus span\{B_r\}.$$

Proof. Since

$$B = B_1 \cup B_2 \cup \cdots \cup B_r$$

and each $B_i \subseteq B$, $(1 \le i \le r)$ we have

$$\text{span}\{B\} = \text{span}\{B_1 \cup B_2 \cup \cdots \cup B_r\} \subseteq \text{span}\{B_1\} \oplus \text{span}\{B_2\} \oplus \cdots \oplus \text{span}\{B_r\} \subseteq \text{span}\{B\}.$$

Therefore

$$\text{span}\{B\} = \text{span}\{B_1\} \oplus \text{span}\{B_2\} \oplus \cdots \oplus \text{span}\{B_r\}.$$

Assume

$$\mathbf{0} = \sum a_{1,j1}\, \boldsymbol{u}_{1,j1} + \sum a_{2,j2}\, \boldsymbol{u}_{2,j2} + \cdots + \sum a_{r,jr}\, \boldsymbol{u}_{r,jr}, \quad (4.\ 24)$$

where $a_{i,ji}$ are scalars and the $\boldsymbol{u}_{i,ji} \in B_i$, $(1 \le i \le r)$. Since B is linearly independent, each $a_{i,ji} = 0$, $(1 \le i \le r)$ in (4. 24). Thus, can only be written uniquely as

$$0 = 0 + 0 + \cdots + 0.$$

Thus,

$$\text{span}\{B\} = \text{span}\{B_1\} \oplus \text{span}\{B_2\} \oplus \cdots \oplus \text{span}\{B_r\}. \qquad \square$$

Proposition 4. 50. *Let*

$$V = U_1 + U_2 + \cdots + U_r$$

be a vector space of the chemical equation (4. 2) *over the field* $\mathbb{R}$ *and let*

$$dimV = dimU_1 + dimU_2 + \cdots + dimU_r,$$

then

$$V = U_1 \oplus U_2 \oplus \cdots \oplus B_r.$$

Proof. Assume dim$V = n$. Let B_i, $(1 \le i \le r)$ be a basis for U_i, $(1 \le i \le r)$. Then

$$B = B_1 \cup B_2 \cup \cdots \cup B_r$$

has n elements and spans V. Thus B is a basis for V. By Proposition 4. 49

$$V = U_1 \oplus U_2 \oplus \cdots \oplus B_r. \qquad \square$$

Theorem 4. 51. *Let V and U be vector spaces over a field* $\mathbb{R}$. *Let* $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ *be a basis of a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *and let* $\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_n$ *be any arbitrary vectors in U, then there exists a unique linear mapping* $F: V \to U$ *such that*

$$F(\boldsymbol{v}_1) = \boldsymbol{u}_1,\ F(\boldsymbol{v}_2) = \boldsymbol{u}_2,\ \ldots,\ F(\boldsymbol{v}_n) = \boldsymbol{u}_n.$$

Proof. Now, we shall define required map $F: V \to U$ such that $F(\boldsymbol{v}_i) = \boldsymbol{u}_i$, $(1 \le i \le n)$. Let $\boldsymbol{v} \in V$. Since $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ is a basis of V, there exist unique scalars $a_i \in \mathbb{R}$, $(1 \le i \le n)$ for which

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n.$$

We define $F: V \to U$ by

$$F(\boldsymbol{v}) = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_n\boldsymbol{u}_n.$$

Since the $a_i \in \mathbb{R}$, $(1 \le i \le n)$ are unique, the mapping F is well-defined.

Now,

$$\boldsymbol{v}_i = 0\boldsymbol{v}_1 + \cdots + 1\boldsymbol{v}_i + \cdots + 0\boldsymbol{v}_n,\ (1 \le i \le n).$$

Therefore

$$F(\boldsymbol{v}_i) = 0\boldsymbol{u}_1 + \cdots + 1\boldsymbol{u}_i + \cdots + 0\boldsymbol{u}_n = \boldsymbol{u}_i.$$

Thus the first step of the proof is completed.

Now, we shall show that F is linear.

Assume

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n$$

and

$$\boldsymbol{w} = b_1\boldsymbol{v}_1 + b_2\boldsymbol{v}_2 + \cdots + b_n\boldsymbol{v}_n.$$

Then

$$\boldsymbol{v} + \boldsymbol{w} = (a_1 + b_1)\boldsymbol{v}_1 + (a_2 + b_2)\boldsymbol{v}_2 + \cdots + (a_n + b_n)\boldsymbol{v}_n$$

and

$$k\boldsymbol{v} = ka_1\boldsymbol{v}_1 + ka_2\boldsymbol{v}_2 + \cdots + ka_n\boldsymbol{v}_n,\ \forall k \in \mathbb{R}.$$

By definition of the mapping F, we have

$$F(\boldsymbol{v}) = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_n\boldsymbol{u}_n$$

and

$$F(\boldsymbol{w}) = b_1\boldsymbol{u}_1 + b_2\boldsymbol{u}_2 + \cdots + b_n\boldsymbol{u}_n.$$

Therefore

$$F(\boldsymbol{v} + \boldsymbol{w}) = (a_1 + b_1)\boldsymbol{u}_1 + (a_2 + b_2)\boldsymbol{u}_2 + \cdots + (a_n + b_n)\boldsymbol{u}_n = (a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_n\boldsymbol{u}_n) + (b_1\boldsymbol{u}_1 + b_2\boldsymbol{u}_2 + \cdots + b_n\boldsymbol{u}_n) = F(\boldsymbol{v}) + F(\boldsymbol{w})$$

and

$$F(k\boldsymbol{v}) = k(a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_n\boldsymbol{u}_n) = kF(\boldsymbol{v}).$$

Thus F is linear.

Now we shall show that F is unique.

Assume $G: V \to U$ is linear and

$$G(\boldsymbol{v}_i) = \boldsymbol{u}_i,\ (1 \le i \le n).$$

If

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n,$$

then

$$G(\boldsymbol{v}) = G(a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n) = a_1G(\boldsymbol{v}_1) + a_2G(\boldsymbol{v}_2) + \cdots + a_nG(\boldsymbol{v}_n) = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_n\boldsymbol{u}_n = F(\boldsymbol{v}).$$

Since $G(\boldsymbol{v}) = F(\boldsymbol{v})$, $\forall \boldsymbol{v} \in V$, $G = F$.

Thus F is unique and the theorem is proved. $\square$

Proposition 4. 52. *If* $F: V \to U$ *is a linear mapping, then kernel of F is a subspace of a* *vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$.

Proof. Since $F(\mathbf{0}) = \mathbf{0}$, $\mathbf{0} \in \mathrm{Ker}F$.

Now, assume $\boldsymbol{v}, \boldsymbol{w} \in \mathrm{Ker}F$ and $a, b \in \mathbb{R}$. Since $\boldsymbol{v}$ and $\boldsymbol{w}$ belong to the kernel of F, $F(\boldsymbol{v}) = \mathbf{0}$ and $F(\boldsymbol{w}) = \mathbf{0}$.

Therefore

$$F(a\boldsymbol{v} + b\boldsymbol{w}) = aF(\boldsymbol{v}) + bF(\boldsymbol{w}) = a\mathbf{0} + b\mathbf{0} = \mathbf{0}$$

and so $a\boldsymbol{v} + b\boldsymbol{w} \in \mathrm{Ker}F$. Thus the kernel of F is a subspace of V. □

Proposition 4. 53. *If* $F: V \to U$ *is a linear mapping, then image of F is a subspace of a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$.

Proof. Since $F(\mathbf{0}) = \mathbf{0}$, $\mathbf{0} \in \mathrm{Im}F$. Now, assume $\boldsymbol{v}, \boldsymbol{w} \in \mathrm{Im}F$ and $a, b \in \mathbb{R}$. Since $\boldsymbol{v}$ and $\boldsymbol{w}$ belong to the image of F, there exist vectors $\boldsymbol{v}', \boldsymbol{w}' \in V$ such that $F(\boldsymbol{v}') = \boldsymbol{v}$ and $F(\boldsymbol{w}') = \boldsymbol{w}$.

Then

$$F(a\boldsymbol{v}' + b\boldsymbol{w}')$$
$$= aF(\boldsymbol{v}') + bF(\boldsymbol{w}') = a\boldsymbol{v} + b\boldsymbol{w} \in \mathrm{Im}V.$$

Thus the image of F is a subspace of V. □

Proposition 4. 54. *If the vectors* $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ *span a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *and* $F: V \to U$ *is a linear mapping, then the vectors* $F(\boldsymbol{v}_1), F(\boldsymbol{v}_2), \ldots, F(\boldsymbol{v}_n) \in U$ *span ImF.*

Proof. Assume $\boldsymbol{u} \in \mathrm{Im}F$, then $F(\boldsymbol{v}) = \boldsymbol{u}$ for some vector $\boldsymbol{v} \in V$. Since $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ span V and since $\boldsymbol{v} \in V$, there exist scalars $a_1, a_2, \ldots, a_n$ for which

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n.$$

Accordingly,

$$\boldsymbol{u} = F(\boldsymbol{v}) = F(a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n)$$
$$= a_1F(\boldsymbol{v}_1) + a_2F(\boldsymbol{v}_2) + \cdots + a_nF(\boldsymbol{v}_n).$$

Thus the vectors $F(\boldsymbol{v}_1), F(\boldsymbol{v}_2), \ldots, F(\boldsymbol{v}_n)$ span ImF. □

Proposition 4. 55. *Let a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *has finite dimension and* $F: V \to U$ *is linear, then ImF has finite dimension, i. e., dim*(*ImF*) ≤ *dimV.*

Proof. Assume $\dim V = n$ and $\dim(\mathrm{Im}F) > \dim V$. Then there exist vectors $\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_{n+1} \in \mathrm{Im}F$ which are linearly independent. Let $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_{n+1}$ be vectors in V such that $F(\boldsymbol{v}_i) = \boldsymbol{w}_i$, $(1 \le i \le n + 1)$.

Assume

$$a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_{n+1}\boldsymbol{v}_{n+1} = \mathbf{0}.$$

Then

$$\mathbf{0} = F(\mathbf{0}) = F(a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_{n+1}\boldsymbol{v}_{n+1})$$
$$= a_1F(\boldsymbol{v}_1) + a_2F(\boldsymbol{v}_2) + \cdots + a_{n+1}F(\boldsymbol{v}_{n+1})$$
$$= a_1\boldsymbol{w}_1 + a_2\boldsymbol{w}_2 + \cdots + a_{n+1}\boldsymbol{w}_{n+1}.$$

Since $\boldsymbol{w}_i$, $(1 \le i \le n + 1)$ are linearly independent, $a_i = 0$, $(1 \le i \le n + 1)$. Thus $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_{n+1}$ are linearly independent.

This contradicts the fact that $\dim V = n$. Thus,

$$\dim(\mathrm{Im}F) \le \dim V. \quad □$$

Theorem 4. 56. *Let a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *has finite dimension and let* $F: V \to U$ *be a linear mapping, then*

$$\dim V = \dim(\mathrm{Ker}F) + \dim(\mathrm{Im}F).$$

Proof. Assume

$$\dim(\mathrm{Ker}F) = r \text{ and } \{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_r\}$$

is a basis of Ker$F$, and assume $\dim(\mathrm{Im}F) = s$ and $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_s\}$ is a basis of ImF. According to the Proposition 4. 55, ImF has finite dimension. Since $\boldsymbol{u}_j \in \mathrm{Im}F$, there exist vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_s$ in V such that

$$F(\boldsymbol{v}_1) = \boldsymbol{u}_1, F(\boldsymbol{v}_2) = \boldsymbol{u}_2, \ldots, F(\boldsymbol{v}_s) = \boldsymbol{u}_s.$$

We claim that the set

$$B = \{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_r, \boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_s\}$$

is a basis of V, *i. e.*, 1º B spans V and 2º B is linearly independent. Once we prove 1º and 2º, then

$$\dim V = r + s = \dim(\mathrm{Ker}F) + \dim(\mathrm{Im}F).$$

1º B spans V. Let $\boldsymbol{v} \in V$. Then $F(\boldsymbol{v}) \in \mathrm{Im}F$. Since the $\boldsymbol{u}_j$ span ImF, there exist scalars a_i, $(1 \le i \le s)$, such that

$$F(\boldsymbol{v}) = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_s\boldsymbol{u}_s.$$

Set

$$\boldsymbol{v}^* = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_s\boldsymbol{v}_s - \boldsymbol{v}.$$

Then

$$F(\boldsymbol{v}^*) = F(a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_s\boldsymbol{v}_s - \boldsymbol{v})$$
$$= a_1F(\boldsymbol{v}_1) + a_2F(\boldsymbol{v}_2) + \cdots + a_sF(\boldsymbol{v}_s) - F(\boldsymbol{v})$$
$$= a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_s\boldsymbol{u}_s - F(\boldsymbol{v}) = \mathbf{0}.$$

Thus $\boldsymbol{v} \in \mathrm{Ker}F$. Since the $\boldsymbol{w}_i$ span Ker$F$, there exist scalars b_i, $(1 \le i \le r)$, such that

$$\boldsymbol{v}^* = b_1\boldsymbol{w}_1 + b_2\boldsymbol{w}_2 + \cdots + b_r\boldsymbol{w}_r$$
$$= a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_s\boldsymbol{v}_s - \boldsymbol{v}.$$

Accordingly,

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_r\boldsymbol{v}_r$$
$$- b_1\boldsymbol{w}_1 - b_2\boldsymbol{w}_2 - \cdots - b_r\boldsymbol{w}_r.$$

Thus B spans V.

2º B is linearly independent. Assume

$$x_1\boldsymbol{w}_1 + x_2\boldsymbol{w}_2 + \cdots + x_r\boldsymbol{w}_r \qquad (4.\ 25)$$
$$+ y_1\boldsymbol{v}_1 + y_2\boldsymbol{v}_2 + \cdots + y_s\boldsymbol{v}_s = \mathbf{0},$$

where $x_i, y_j \in \mathbb{R}$, $(1 \le i \le r, 1 \le j \le s)$.

Then

$$\mathbf{0} = F(\mathbf{0}) = \mathrm{F}(x_1\boldsymbol{w}_1 + x_2\boldsymbol{w}_2 + \cdots + x_r\boldsymbol{w}_r \quad (4.26)$$
$$+ y_1\boldsymbol{v}_1 + y_2\boldsymbol{v}_2 + \cdots + y_s\boldsymbol{v}_s)$$
$$= x_1F(\boldsymbol{w}_1) + x_2F(\boldsymbol{w}_2) + \cdots + x_rF(\boldsymbol{w}_r)$$
$$+ y_1F(\boldsymbol{v}_1) + y_2F(\boldsymbol{v}_2) + \cdots + y_sF(\boldsymbol{v}_s).$$

But $F(\boldsymbol{w}_i) = \mathbf{0}$, $(1 \le i \le r)$ since $\boldsymbol{w}_i \in \mathrm{Ker}F$, $(1 \le i \le r)$ and $F(\boldsymbol{v}_j) = \boldsymbol{u}_j$, $(1 \le j \le s)$. Substitution in (4. 26) gives $y_1\boldsymbol{u}_1 + y_2\boldsymbol{u}_2 + \cdots + y_s\boldsymbol{u}_s = \mathbf{0}$. Since the $\boldsymbol{u}_j$, $(1 \le j \le s)$ are linearly independent, each $y_j = 0$, $(1 \le j \le s)$. Substitution in (4. 25) gives

$$x_1\boldsymbol{w}_1 + x_2\boldsymbol{w}_2 + \cdots + x_r\boldsymbol{w}_r = \mathbf{0}.$$

Since the $\boldsymbol{w}_i$, $(1 \le i \le r)$ are linearly independent, each $x_i = 0$, $(1 \le i \le r)$. Thus B is linearly independent. By this the proof is completed. □

Proposition 4. 57. *Let a vector space V of the chemical equation* (4. 2) *over the field* ℝ *has finite dimension and let F*: *V* → *U be a linear mapping, then V and the image of F have the same dimension if only if F is nonsingular.*

Proof. By Theorem 4. 56, $\dim V = \dim(\mathrm{Im}F) + \dim(\mathrm{Ker}F)$. Therefore V and $\mathrm{Im}F$ have the same dimension if and only if $\dim(\mathrm{Ker}F) = 0$ or $\mathrm{Ker}F = \{\mathbf{0}\}$, *i. e.*, if and only if F is nonsingular. □

Proposition 4. 58. *Let a vector space V of the chemical equation* (4. 2) *over the field* ℝ *has finite dimension and let F*: *V* → *U be a linear mapping, such that maps independent sets into independent sets, then F is nonsingular.*

Proof. Assume $\boldsymbol{v} \in V$ is nonzero vector, then $\{\boldsymbol{v}\}$ is independent. Then $\{F(\boldsymbol{v})\}$ is independent and so $F(\boldsymbol{v}) \ne \mathbf{0}$.

Therefore, F is nonsingular. □

Theorem 4. 59. *Let a vector space V of the chemical equation* (4. 2) *over the field* ℝ *has finite dimension and let F*: *V* → *U be a linear mapping, then the image of any linearly independent set is linearly independent.*

Proof. Assume $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ are linearly independent vectors in V. We claimed that the vectors $F(\boldsymbol{v}_1), F(\boldsymbol{v}_2), \ldots, F(\boldsymbol{v}_n)$ are also linearly independent.

Assume

$$a_1F(\boldsymbol{v}_1) + a_2F(\boldsymbol{v}_2) + \cdots + a_nF(\boldsymbol{v}_n) = \mathbf{0},$$

where $a_i \in \mathbb{R}$, $(1 \le i \le n)$.

Since F is linear,

$$F(a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n) = \mathbf{0},$$

therefore $a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n$ belongs to $\mathrm{Ker}F$. But F is nonsingular, *i. e.*, $\mathrm{Ker}F = \{\mathbf{0}\}$, therefore

$$a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n = \mathbf{0}.$$

Since the $\boldsymbol{v}_i$ $(1 \le i \le n)$ are linearly independent, all the a_i are 0. Accordingly, the $F(\boldsymbol{v}_i)$, $(1 \le i \le n)$ are linearly independent. Accordingly the theorem is proved. □

Proposition 4. 60. *Let a vector space V of the chemical equation* (4. 2) *over the field* ℝ *has finite dimension and dimV = dimU, then a linear mapping F*: *V* → *U is nonsingular if and only if F is surjective, i. e., maps V onto U.*

Proof. By Theorem 4. 56,

$\dim V = \dim(\mathrm{Ker}F) + \dim(\mathrm{Im}F)$.

Thus F is surjective if and only if

$$\mathrm{Im}F = U$$

if and only if

$$\dim(\mathrm{Im}F) = \dim U = \dim V$$

if and only if

$$\dim(\mathrm{Ker}F) = 0$$

if and only if F is nonsingular. □

Proposition 4. 61. *Let a vector space V of the chemical equation* (4. 2) *over the field* ℝ *has dimV = n, then* $V \simeq \mathbb{R}^n$.

Proof. Let $\{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_n\}$ be a basis of V. Let $[\boldsymbol{v}]$ denote the coordinates of $\boldsymbol{v} \in V$ relative to the given basis. Then the map $F: V \to \mathbb{R}^n$ defined by $F(\boldsymbol{v}) = [\boldsymbol{v}]$ is an isomorphism. Thus $V \simeq \mathbb{R}^n$. □

Theorem 4. 62. *Let a vector space V of the chemical equation* (4. 2) *over the field* ℝ *has finite dimension, such that dimV = dimU and let F*: *V* → *U be a linear mapping, then F is an isomorphism if and only if F is nonsingular.*

Proof. If F is an isomorphism then only 0 maps onto 0 so F is nonsingular.

Assume F is nonsingular. Then $\dim(\mathrm{Ker}F) = 0$.

By Theorem 4. 56,

$$\dim V = \dim(\mathrm{Ker}F) + \dim(\mathrm{Im}F).$$

Therefore,

$$\dim U = \dim V = \dim(\mathrm{Im}F).$$

Since U has finite dimension, $\mathrm{Im}F = U$ and so F is surjective. Thus F is both one-to-one and onto, *i. e.*, F is an isomorphism. □

Proposition 4. 63. *The orthogonal complement* $W^{\perp}$ *is a subspace of a vector space V of the chemical equation* (4. 2) *over the field* ℝ.

Prof. Obviously, $0 \in W^{\perp}$. Now, we assume $\boldsymbol{u}, \boldsymbol{v} \in W^{\perp}$. Then $\forall a, b \in \mathbb{R} \wedge \forall \boldsymbol{w} \in W$,

$$\langle a\boldsymbol{u} + b\boldsymbol{v}, \boldsymbol{w}\rangle = a\langle \boldsymbol{u}, \boldsymbol{w}\rangle + b\langle \boldsymbol{v}, \boldsymbol{w}\rangle$$
$$= a\cdot 0 + b\cdot 0 = 0.$$

Therefore, $a\boldsymbol{u} + b\boldsymbol{v} \in W^{\perp}$ and thus W is a subspace of V. □

Proposition 4. 64. *If the set of vectors* $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ *of the chemical equations* (4. 2) *is orthogonal, then the vectors* v_i, $(1 \le i \le n)$ *are linearly independent.*

Proof. Assume $S = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ and assume

$$a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n = \boldsymbol{0}. \quad (4.\ 27)$$

Taking the inner product of (4. 27) with $\boldsymbol{v}_1$ we get

$$0 = \langle \boldsymbol{0}, \boldsymbol{v}_1\rangle = \langle a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n, \boldsymbol{v}_1\rangle$$
$$= a_1\langle \boldsymbol{v}_1, \boldsymbol{v}_1\rangle + a_2\langle \boldsymbol{v}_2, \boldsymbol{v}_1\rangle + \cdots + a_n\langle \boldsymbol{v}_n, \boldsymbol{v}_1\rangle$$
$$= a_1\langle \boldsymbol{v}_1, \boldsymbol{v}_1\rangle + a_2\cdot 0 + \cdots + a_n\cdot 0 = a_1\langle \boldsymbol{v}_1, \boldsymbol{v}_1\rangle.$$

Since S is orthogonal, $\langle \boldsymbol{v}_1, \boldsymbol{v}_1\rangle \neq 0$, therefore $a_1 = 0$. Similarly, for $2 \le i \le n$ taking the inner product of (4. 27) with $\boldsymbol{v}_i$,

$$0 = \langle \boldsymbol{0}, \boldsymbol{v}_i\rangle = \langle a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n, \boldsymbol{v}_i\rangle$$
$$= a_1\langle \boldsymbol{v}_1, \boldsymbol{v}_i\rangle + \cdots + a_i\langle \boldsymbol{v}_i, \boldsymbol{v}_i\rangle + \cdots + a_n\langle \boldsymbol{v}_n, \boldsymbol{v}_i\rangle$$
$$= a_i\langle \boldsymbol{v}_i, \boldsymbol{v}_i\rangle.$$

But, $\langle \boldsymbol{v}_i, \boldsymbol{v}_i\rangle \neq 0$ and therefore $a_i = 0$, $(1 \le i \le n)$.

Thus S is linearly independent. □

Proposition 4. 65. *If the set of vectors* $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ *of the chemical equations* (4. 2) *is orthogonal, then* $\{a_1\boldsymbol{v}_1, a_2\boldsymbol{v}_2, \ldots, a_n\boldsymbol{v}_n\}$ *is orthogonal* $\forall a_i \in \mathbb{R}$, $a_i \neq 0$, $(1 \le i \le n)$.

Proof. Since $\boldsymbol{v}_i \neq \boldsymbol{0}$, $(1 \le i \le n)$ and $a_i \neq 0$, $(1 \le i \le n)$, we obtain $a_i\boldsymbol{v}_i \neq 0$, $(1 \le i \le n)$. Also, for $i \neq j$, $\langle \boldsymbol{v}_i, \boldsymbol{v}_j\rangle = 0$ and therefore

$$\langle a_i\boldsymbol{v}_i, a_j\boldsymbol{v}_j\rangle = a_ia_j\cdot\langle \boldsymbol{v}_i, \boldsymbol{v}_j\rangle = a_ia_j\cdot 0 = 0.$$

Thus $\{a_1\boldsymbol{v}_1, a_2\boldsymbol{v}_2, \ldots, a_n\boldsymbol{v}_n\}$ is orthogonal. □

Next, we shall continue this section, with a practical approach regarding the tobacco combustion process. The text which follows, it provides a comprehensive description about that process.

Tobacco is a plant substance which contains approximately 3800 components, ranging from small organic and inorganic molecules to biopolymers [23].

The small molecules belong to various classes of compounds such as acids, alcohols, aldehydes, alkaloids, amino acids, Amadori compounds, carbohydrates, esters, isoprenoids, ketones, nitriles, phenols, quinones, sterols, sulphur compounds, terpenes etc.

The biopolymers contain cellulose, hemicellulose, lignin, nucleic acids, pectin, proteins and peptides, starch, etc. During cigarette smoking, these are all subjected to temperatures up to 950ºC in the presence of varying concentrations of oxygen in the burning zone or other tobacco product [24]. According to the research [25] about 4800 substances in tobacco smoke are identified. Many of the smoke components occur from different phases including distillation from tobacco, combustion, pyrolysis and pyrosynthetic reactions. In tobacco smoke are detected 2800 components, indicating the importance of pyrolysis, pyrosynthetic and combustion formation mechanisms.

Rough structural relations of the main combustion processes involved in smoke can be interpreted by the following block diagram shown by Fig. 1.

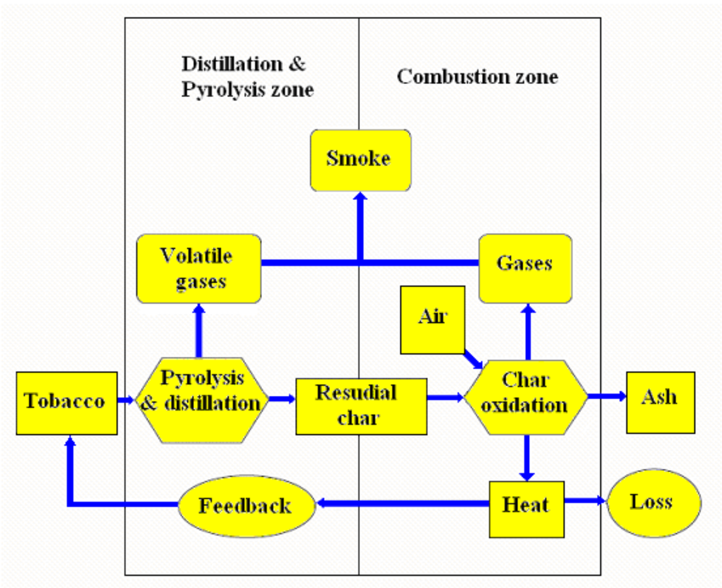

Fig. 1. *A block diagram of cigarette combustion*

In [26] is considered the combustion process and approximately are given the conditions inside the burning zone of the cigarette that influence product formation mechanisms. These include determination of the temperature and gas formation contour distributions at various times in the smoking cycle of a burning cigarette.

A graphic illustration of the major smoke formation mechanisms occurring inside the cigarette is shown by Fig. 2.

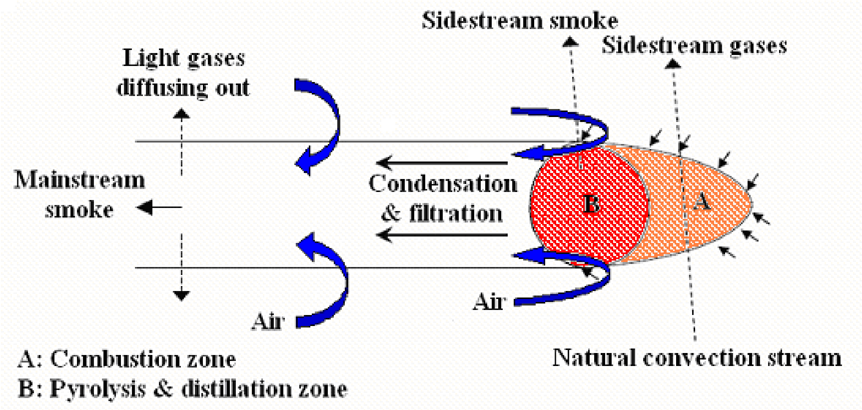

Fig. 2. *The burning cigarette during a puff*

The internal part of the burning region, which has deficit of oxygen and surplus of hydrogen, can be effectively divided into two zones:

A: an exothermic combustion zone, and
B: an endothermic pyrolysis/distillation zone.

Like air is drawn into the cigarette during the puff, oxygen is consumed by combustion with carbonized tobacco and the simple combustion products carbon monoxide, carbon dioxide and water are produced, together with the release of heat which sustains the whole burning process.

Temperature in this region is generated between 700 and 950ºC, and heating rate as high as 500ºC/s is achieved. Instantly downstream of the combustion region is the pyrolysis/distillation zone, where the temperature is between 200 and 600ºC and which is still low in oxygen. Most of smoke products are generated in this region by a variety of mechanisms which are essentially endothermic. A highly concentrated, perhaps supersaturated, vapor is generated and, during a puff, is drawn down the tobacco rod to constitute the mainstream smoke. Its dwelling time in the formation region is very short, only a few milliseconds.

Like the generated vapor is drawn out of the pyrolysis/distillation region during the puff, it cools very rapidly in the presence of diluting air entering at the paper burn line. This brings the vapors of the less volatile compounds rapidly to their infiltration point and condensation takes place like the vapor cools below 350ºC. A dense aerosol consisting of growing droplet particles is produced. The mainstream smoke emerging from the mouth end of the cigarette during a puff is a highly concentrated, dynamic aerosol system. The smoke particles are liquid, with water making up to 20% of the droplet volume and, consequently, they have spherical forms.

Also, the size of smoke particles will increase in humid environments due to absorption of water vapor. This is particularly important in the respiratory tract, where the relative humidity is 99.5%. The relative humidity of mainstream smoke is 60-70% and little particle growth due to water absorption occurs below 90% relative humidity. Particle growth increases sharply with humidity above 90% due to the absorption of water and smoke particles double in size at 99.5% relative humidity. Smoke particle growth by absorption is a complex process dependent on the chemical composition of both the particles and surrounding gas phase. It occurs in milliseconds.

CO and CO_2 are the main products in cigarette smoke and are produced by both thermal decomposition and combustion of many of the components of tobacco: amino acids, carboxylic acids, esters, celluloses, sugars, starch etc. Studies in which the cigarette was smoked or pyrolysed in isotopically labeled oxygen and carbon dioxide [27] have shown that approximately 30% of the carbon monoxide is formed by thermal decomposition of tobacco constituents, about 36% by combustion of tobacco and at least 23% by the endothermic carbonaceous reduction of carbon dioxide: $C + CO_2 \rightarrow 2CO$.

Cigarette smoke is a complex mixture of 4800 identified substances, and 45 are believed by Regulatory Authorities in Canada and the USA to be relevant to smoking-related diseases [28].

If we take into account these substances, together with their approximate amounts in cigarette smoke and their phase in smoke, and phytochemicals that cultivated tobacco or *Nicotiana tabacum* contains, then the following reaction of tobacco combustion will take a place

$$x_1\, C_5H_{14}NO + x_2\, C_8H_{10}N_2O \qquad (4.\ 28)$$
$$+ x_3\, C_{23}H_{32}N_2O_3 + x_4\, C_4H_9N + x_5\, C_9H_{10}N_2$$
$$+ x_6\, C_9H_{12}N_2 + x_7\, C_{10}H_{12}N_2 + x_8\, C_{10}H_{14}N_2$$
$$+ x_9\, C_{15}H_{13}N_3 + x_{10}\, C_3H_4O + x_{11}\, C_3H_6O_2$$
$$+ x_{12}\, C_7H_6O_2 + x_{13}\, C_{10}H_{12}O + x_{14}\, C_{13}H_{12}O_2$$
$$+ x_{15}\, C_{14}H_{10}O_3 + x_{16}\, C_{16}H_{18}O_9 + x_{17}\, C_{21}H_{20}O_{12}$$
$$+ x_{18}\, C_{25}H_{30}O_{15} + x_{19}\, C_{27}H_{30}O_{16} + x_{20}\, O_2$$
$$\rightarrow x_{21}\, C_5H_5N + x_{22}\, C_9H_7N + x_{23}\, C_{10}H_7N$$
$$+ x_{24}\, C_{10}H_9N + x_{25}\, C_{15}H_{17}N_3 + x_{26}\, C_6H_5NO_2$$
$$+ x_{27}\, C_{10}H_{14}N_2O + x_{28}\, CH_2O + x_{29}\, C_2H_4O$$
$$+ x_{30}\, C_3H_4O + x_{31}\, C_3H_6O + x_{32}\, C_4H_6O$$
$$+ x_{33}\, C_4H_8O + x_{34}\, C_6H_6O_2 + x_{35}\, C_{28}H_{48}O$$
$$+ x_{36}\, C_{29}H_{48}O + x_{37}\, C_{29}H_{50}O + x_{38}\, HCN$$
$$+ x_{39}\, C_4H_6 + x_{40}\, C_5H_8 + x_{41}\, C_8H_8 + x_{42}\, C_{16}H_{10}$$
$$+ x_{43}\, C_{20}H_{32} + x_{44}\, CO + x_{45}\, CO_2 + x_{46}\, NO$$
$$+ x_{47}\, NO_2 + x_{48}\, NH_3 + x_{49}\, H_2O.$$

Here, the author would like to emphasize very clearly that the organic reaction (4. 28), which describes the tobacco combustion, is a completely new reaction and for the first time it appears in scientific literature. Until now, the tobacco combustion was described by some elementary chemical reactions, which did not satisfy accurately real conditions, and they provided only a rough picture about combustion zone.

Since we have all necessary preconditions for balancing a chemical reaction, now we shall balance the main reaction of tobacco combustion.

This reaction includes all important alkaloids and toxins which tobacco contains.

The reaction (4. 28) belongs to the class of continuum reactions and its balancing is a tough question, which enters deeply in the theory of linear programming, because the necessary and sufficient conditions generate very hard solvable systems of linear inequalities in several variables.

Since, the reaction (4.28) has a stoichiometric matrix

$$\boldsymbol{A} = \begin{bmatrix} 5 & 8 & 23 & 4 & 9 & 9 & 10 & 10 & 15 & 3 & 3 & 7 & 10 & 13 & 14 & 16 & 21 & 25 & 27 & 0 & -5 & -9 & -10 & -10 & -15 & -6 & -10 & -1 & -2 & -3 & -3 & -4 & -4 & -6 & -28 & -29 & -29 & -1 & -4 & -5 & -8 & -16 & -20 & -1 & -1 & 0 & 0 & 0 & 0 \\ 14 & 10 & 32 & 9 & 10 & 12 & 12 & 14 & 13 & 4 & 6 & 6 & 12 & 12 & 10 & 18 & 20 & 30 & 30 & 0 & -5 & -7 & -7 & -9 & -17 & -5 & -14 & -2 & -4 & -4 & -6 & -6 & -8 & -6 & -48 & -48 & -50 & -1 & -6 & -8 & -8 & -10 & -32 & 0 & 0 & 0 & 0 & -3 & -2 \\ 1 & 2 & 2 & 1 & 2 & 2 & 2 & 2 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -3 & -1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 2 & 3 & 9 & 12 & 15 & 16 & 2 & 0 & 0 & 0 & 0 & 0 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -2 & -1 & -2 & 0 & -1 \end{bmatrix},$$

with a rank$\boldsymbol{A}$ = 4 and 49 molecules, it can be balanced on $49!/[4!(49 - 4)!] = 211876$ different ways. In other words, there are 211876 general solutions, different each other, such that every one generates an infinite number of particular solutions. Only on this way can be fully balanced reaction (4. 28). Really, it is an extremely hard problem. In this work we shall find only one general solution and one particular solution.

The above chemical reaction (4. 28) reduces to the following system of linear equations

$$5x_1 + 8x_2 + 23x_3 + 4x_4 + 9x_5 + 9x_6 + 10x_7 + 10x_8$$
$$+ 15x_9 + 3x_{10} + 3x_{11} + 7x_{12} + 10x_{13} + 13x_{14}$$
$$+ 14x_{15} + 16x_{16} + 21x_{17} + 25x_{18} + 27x_{19} = 5x_{21}$$
$$+ 9x_{22} + 10x_{23} + 10x_{24} + 15x_{25} + 6x_{26} + 10x_{27}$$
$$+ x_{28} + 2x_{29} + 3x_{30} + 3x_{31} + 4x_{32} + 4x_{33} + 6x_{34}$$
$$+ 28x_{35} + 29x_{36} + 29x_{37} + x_{38} + 4x_{39}$$
$$+ 5x_{40} + 8x_{41} + 16x_{42} + 20x_{43} + x_{44} + x_{45},$$

$$14x_1 + 10x_2 + 32x_3 + 9x_4 + 10x_5 + 12x_6 + 12x_7 + 14x_8 + 13x_9 + 4x_{10} + 6x_{11} + 6x_{12} + 12x_{13} + 12x_{14} + 10x_{15} + 18x_{16} + 20x_{17} + 30x_{18} + 30x_{19} = 5x_{21} + 7x_{22} + 7x_{23} + 9x_{24} + 17x_{25} + 5x_{26} + 14x_{27} + 2x_{28} + 4x_{29} + 4x_{30} + 6x_{31} + 6x_{32} + 8x_{33} + 6x_{34} + 48x_{35} + 48x_{36} + 50x_{37} + x_{38} + 6x_{39} + 8x_{40} + 8x_{41} + 10x_{42} + 32x_{43} + 3x_{48} + 2x_{49},$$

$$x_1 + 2x_2 + 2x_3 + x_4 + 2x_5 + 2x_6 + 2x_7 + 2x_8 + 3x_9 = x_{21} + x_{22} + x_{23} + x_{24} + 3x_{25} + x_{26} + 2x_{27} + x_{38} + x_{46} + x_{47} + x_{48}, \qquad (4.\ 29)$$

$$x_1 + x_2 + 3x_3 + x_{10} + 2x_{11} + 2x_{12} + x_{13} + 2x_{14} + 3x_{15} + 9x_{16} + 12x_{17} + 15x_{18} + 16x_{19} + 2x_{20} = 2x_{26} + x_{27} + x_{28} + x_{29} + x_{30} + x_{31} + x_{32} + x_{33} + 2x_{34} + x_{35} + x_{36} + x_{37} + x_{44} + 2x_{45} + x_{46} + 2x_{47} + x_{49}.$$

The general solution of the system (4.29) is given by the following expressions

$$x_1 = (158x_5 + 128x_6 + 166x_7 + 136x_8 + 324x_9 - 66x_{10} - 216x_{11} - 64x_{12} + 80x_{13} + 74x_{14} + 22x_{15} - 742x_{16} - 942x_{17} - 1300x_{18} - 1344x_{19} - 240x_{20} - 98x_{21} - 220x_{22} - 258x_{23} - 228x_{24} - 264x_{25} + 104x_{26} - 16x_{27} + 112x_{28} + 104x_{29} + 66x_{30} + 96x_{31} + 58x_{32} + 88x_{33} + 102x_{34} - 224x_{35} - 262x_{36} - 232x_{37} - 6x_{38} - 62x_{39} - 70x_{40} - 184x_{41} - 458x_{42} - 280x_{43} + 82x_{44} + 202x_{45} + 137x_{46} + 257x_{47} + 62x_{48} + 150x_{49})/157,$$

$$x_2 = (- 95x_5 - 71x_6 - 70x_7 - 46x_8 - 165x_9 - 10x_{10} - 47x_{11} - 43x_{12} + 93x_{13} + 35x_{14} - 49x_{15} - 317x_{16} - 471x_{17} - 530x_{18} - 589x_{19} - 122x_{20} + 47x_{21} + 19x_{22} + 18x_{23} - 6x_{24} + 117x_{25} + 168x_{26} + 107x_{27} + 36x_{28} + 11x_{29} + 10x_{30} - 14x_{31} - 15x_{32} - 39x_{33} + 44x_{34} - 543x_{35} - 544x_{36} - 568x_{37} + 99x_{38} - 76x_{39} - 101x_{40} - 104x_{41} - 136x_{42} - 404x_{43} + 60x_{44} + 121x_{45} + 173x_{46} + 234x_{47} + 76x_{48} + 37x_{49})/157, \qquad (4.\ 30)$$

$$x_3 = (21x_5 - 19x_6 - 32x_7 - 30x_8 - 53x_9 - 27x_{10} - 17x_{11} - 69x_{12} - 110x_{13} - 141x_{14} - 148x_{15} - 118x_{16} - 157x_{17} - 175x_{18} - 193x_{19} + 16x_{20} + 17x_{21} + 67x_{22} + 80x_{23} + 78x_{24} + 49x_{25} + 14x_{26} + 22x_{27} + 3x_{28} + 14x_{29} + 27x_{30} + 25x_{31} + 38x_{32} + 36x_{33} + 56x_{34} + 308x_{35} + 321x_{36} + 319x_{37} - 31x_{38} + 46x_{39} + 57x_{40} + 96x_{41} + 198x_{42} + 228x_{43} + 5x_{44} - 3x_{45} - 51x_{46} - 59x_{47} - 46x_{48} - 10x_{49})/157,$$

$$x_4 = (- 240x_5 - 262x_6 - 276x_7 - 298x_8 - 359x_9 + 140x_{10} + 344x_{11} + 288x_{12} - 46x_{13} + 138x_{14} + 372x_{15} + 1612x_{16} + 14x_{17} + 2710x_{18} + 2908x_{19} + 452x_{20} + 127x_{21} + 205x_{22} + 219x_{23} + 241x_{24} + 403x_{25} - 311x_{26} + 72x_{27} - 190x_{28} - 154x_{29} - 140x_{30} - 118x_{31} - 104x_{32} - 82x_{33} - 302x_{34} + 694x_{35} + 708x_{36} + 730x_{37} + 27x_{38} + 122x_{39} + 158x_{40} + 200x_{41} + 334x_{42} + 632x_{43} - 212x_{44} - 438x_{45} - 224x_{46} - 450x_{47} + 35x_{48} - 204x_{49})/157,$$

where x_i, $(5 \le i \le 49)$ are arbitrary real numbers.

Balanced reaction (4. 28) has four generators

$$x_i > 0,\ (1 \le i \le 4), \qquad (4.\ 31)$$

which produce an infinite number of particular solutions.

From (4. 31) we obtain the following system of linear inequalities

$$158x_5 + 128x_6 + 166x_7 + 136x_8 + 324x_9 - 66x_{10} - 216x_{11} - 64x_{12} + 80x_{13} + 74x_{14} + 22x_{15} - 742x_{16} - 942x_{17} - 1300x_{18} - 1344x_{19} - 240x_{20} - 98x_{21} - 220x_{22} - 258x_{23} - 228x_{24} - 264x_{25} + 104x_{26} - 16x_{27} + 112x_{28} + 104x_{29} + 66x_{30} + 96x_{31} + 58x_{32} + 88x_{33} + 102x_{34} - 224x_{35} - 262x_{36} - 232x_{37} - 6x_{38} - 62x_{39} - 70x_{40} - 184x_{41} - 458x_{42} - 280x_{43} + 82x_{44} + 202x_{45} + 137x_{46} + 257x_{47} + 62x_{48} + 150x_{49} > 0,$$

$$- 95x_5 - 71x_6 - 70x_7 - 46x_8 - 165x_9 - 10x_{10} - 47x_{11} - 43x_{12} + 93x_{13} + 35x_{14} - 49x_{15} - 317x_{16} - 471x_{17} - 530x_{18} - 589x_{19} - 122x_{20} + 47x_{21} + 19x_{22} + 18x_{23} - 6x_{24} + 117x_{25} + 168x_{26} + 107x_{27} + 36x_{28} + 11x_{29} + 10x_{30} - 14x_{31} - 15x_{32} - 39x_{33} + 44x_{34} - 543x_{35} - 544x_{36} - 568x_{37} + 99x_{38} - 76x_{39} - 101x_{40} - 104x_{41} - 136x_{42} - 404x_{43} + 60x_{44} + 121x_{45} + 173x_{46} + 234x_{47} + 76x_{48} + 37x_{49} > 0, \qquad (4.\ 32)$$

$$21x_5 - 19x_6 - 32x_7 - 30x_8 - 53x_9 - 27x_{10} - 17x_{11} - 69x_{12} - 110x_{13} - 141x_{14} - 148x_{15} - 118x_{16} - 157x_{17} - 175x_{18} - 193x_{19} + 16x_{20} + 17x_{21} + 67x_{22} + 80x_{23} + 78x_{24} + 49x_{25} + 14x_{26} + 22x_{27} + 3x_{28} + 14x_{29} + 27x_{30} + 25x_{31} + 38x_{32} + 36x_{33} + 56x_{34} + 308x_{35} + 321x_{36} + 319x_{37} - 31x_{38} + 46x_{39} + 57x_{40} + 96x_{41} + 198x_{42} + 228x_{43} + 5x_{44} - 3x_{45} - 51x_{46} - 59x_{47} - 46x_{48} - 10x_{49} > 0,$$

$$- 240x_5 - 262x_6 - 276x_7 - 298x_8 - 359x_9 + 140x_{10} + 344x_{11} + 288x_{12} - 46x_{13} + 138x_{14} + 372x_{15} + 1612x_{16} + 14x_{17} + 2710x_{18} + 2908x_{19} + 452x_{20} + 127x_{21} + 205x_{22} + 219x_{23} + 241x_{24} + 403x_{25} - 311x_{26} + 72x_{27} - 190x_{28} - 154x_{29} - 140x_{30} - 118x_{31} - 104x_{32} - 82x_{33} - 302x_{34} + 694x_{35} + 708x_{36} + 730x_{37} + 27x_{38} + 122x_{39} + 158x_{40} + 200x_{41} + 334x_{42} + 632x_{43} - 212x_{44} - 438x_{45} - 224x_{46} - 450x_{47} + 35x_{48} - 204x_{49} > 0$$

The inequalities (4. 32) are necessary and suficient conditions to hold (4. 28). In other words (4. 20) holds if and only if are satisfied (4. 32)

If we take into account (4. 30) immediately follows balanced reaction

$$20985498810722\ C_5H_{14}NO \qquad (4.33)$$
$$+ 211988664173123\ C_8H_{10}N_2O$$
$$+ 71404734962373\ C_{23}H_{32}N_2O_3$$
$$+ 74612231873620\ C_4H_9N$$
$$+ 64075324647690\ C_9H_{10}N_2$$
$$+ 66410485874690\ C_9H_{12}N_2$$
$$+ 67029087720750\ C_{10}H_{12}N_2$$
$$+ 69364248947750\ C_{10}H_{14}N_2$$

$$+ 56492180494820\ C_{15}H_{13}N_3$$
$$+ 74139764219020\ C_3H_4O$$
$$+ 67661465635250\ C_3H_6O_2$$
$$+ 70135873019490\ C_7H_6O_2$$
$$+ 87997222049440\ C_{10}H_{12}O$$
$$+ 80852967776850\ C_{13}H_{12}O_2$$
$$+ 65431891883276\ C_{14}H_{10}O_3$$
$$+ 28020038320640\ C_{16}H_{18}O_9$$
$$+ 7007829345630\ C_{21}H_{20}O_{12}$$
$$+ 5282336567440\ C_{25}H_{30}O_{15}$$
$$+ 6429296343045\ C_{27}H_{30}O_{16}$$
$$+ 41657297999398\ O_2$$
$$\rightarrow 81971892630682\ C_5H_5N$$
$$+ 76226627132560\ C_9H_7N$$
$$+ 73010292315826\ C_{10}H_7N$$
$$+ 74740831521832\ C_{10}H_9N$$
$$+ 93344031289188\ C_{15}H_{17}N_3$$
$$+ 206607536035418\ C_6H_5NO_2$$
$$+ 94371998670282\ C_{10}H_{14}N_2O$$
$$+ 80908579479454\ CH_2O$$
$$+ 79543512098852\ C_2H_4O$$
$$+ 78225105368756\ C_3H_4O$$
$$+ 77436648403356\ C_3H_6O$$
$$+ 74952028932304\ C_4H_6O$$
$$+ 74125308440176\ C_4H_8O$$
$$+ 80627882148850\ C_6H_6O_2$$
$$+ 16837231445704\ C_{28}H_{48}O$$
$$+ 11981945330022\ C_{29}H_{48}O$$
$$+ 9621876521676\ C_{29}H_{50}O$$
$$+ 87674677063278\ HCN$$
$$+ 66578882188196\ C_4H_6$$
$$+ 64203924295902\ C_5H_8$$
$$+ 60669668898798\ C_8H_8$$
$$+ 54853660365650\ C_{16}H_{10}$$
$$+ 26692479484410\ C_{20}H_{32}$$
$$+ 84621954002320\ CO$$
$$+ 93435413813090\ CO_2$$
$$+ 100038052918110\ NO$$
$$+ 108851512728880\ NO_2$$
$$+ 87721851266840\ NH_3$$
$$+ 82905394621380\ H_2O.$$

The system (4. 29) has four (nonzero) linear equations in fourty-nine unknowns; and hence it has 49 - 4 = 45 free variables $x_i > 0$, $(5 \leq i \leq 49)$. Thus, the dimension of the solution space W of the system (4. 29) is dim $W = 45$. To obtain a basis for W, we set

$$x_5 = 1,\ x_6 = \cdots = x_{49} = 0,$$
$$x_5 = 0,\ x_6 = 1,\ x_7 = \cdots = x_{49} = 0,$$
$$x_5 = x_6 = 0,\ x_7 = 1,\ x_8 = \cdots = x_{49} = 0, \quad (4.\ 34)$$
$$\vdots$$
$$x_5 = \cdots = x_{47} = 0,\ x_{48} = 1,\ x_{49} = 0,$$
$$x_5 = \cdots = x_{48} = 0,\ x_9 = 1,$$

in the expressions (4. 30) to obtain the solutions

$\boldsymbol{a}_1$ = (158/157, -95/157, 21/157, -240/157, 1, 0, 0, …, 0, 0),

$\boldsymbol{a}_2$ = (128/157, -71/157, -19/157, -262/157, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_3$ = (166/157, -70/157, -32/157, -276/157, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_4$ = (136/157, -46/157, -30/157, -298/157, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_5$ = (324/157, -165/157, -53/157, -359/157, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_6$ = (-66/157, -10/157, -27/157, 140/157, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_7$ = (-216/157, -47/157, -17/157, 344/157, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_8$ = (-64/157, -43/157, -69/157, 288/157, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_9$ = (80/157, 93/157, -110/157, -46/157, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{10}$ = (74/157, 35/157, -141/157, 138/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{11}$ = (22/157, -49/157, -148/157, 372/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{12}$ = (-742/157, -317/157, -118/157, 1612/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{13}$ = (-942/157, -471/157, -157/157, 14/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{14}$ = (-1300/157, -530/157, -175/157, 2710/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{15}$ = (-1344/157, -589/157, -193/157, 2908/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{16}$ = (-240/157, -122/157, 16/157, 452/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{17}$ = (-98/157, 47/157, 17/157, 127/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{18}$ = (-220/157, 19/157, 67/157, 205/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{19}$ = (-258/157, 18/157, 80/157, 219/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{20}$ = (-228/157, -6/157, 78/157, 241/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{21}$ = (-264/157, 117/157, 49/157, 403/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{22}$ = (104/157, 168/157, 14/157, -311/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{23}$ = (-16/157, 107/157, 22/157, 72/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{24}$ = (112/157, 36/157, 3/157, -190/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{25}$ = (104/157, 11/157, 14/157, -154/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{26}$ = (66/157, 10/157, 27/157, -140/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{27}$ = (96/157, -14/157, 25/157, -118/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{28}$ = (58/157, -15/157, 38/157, -104/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{29}$ = (88/157, -39/157, 36/157, -82/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{30}$ = (102/157, 44/157, 56/157, -302/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{31}$ = (-224/157, -543/157, 308/157, 694/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{32}$ = (-262/157, -544/157, 321/157, 708/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{33}$ = (-232/157, -568/157, 319/157, 730/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{34}$ = (-6/157, 99/157, -31/157, 27/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{35}$ = (-62/157, -76/157, 46/157, 122/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{36}$ = (-70/157, -101/157, 57/157, 158/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{37}$ = (-184/157, -104/157, 96/157, 200/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{38}$ = (-458/157, -136/157, 198/157, 334/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{39}$ = (-280/157, -404/157, 228/157, 632/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{40}$ = (82/157, 60/157, 5/157, 212/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{41}$ = (202/157, 121/157, -3/157, -438/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{42}$ = (137/157, 173/157, -51/157, -224/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0),

$\boldsymbol{a}_{43}$ = (257/157, 234/157, -59/157, -450/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0),

$\boldsymbol{a}_{44}$ = (62/157, 76/157, -46/157, 35/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,0),

$\boldsymbol{a}_{45}$ = (150/157, 37/157, -10/157, -204/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1),

The set $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_{45}\}$ is a basis of the solution space W.

The reaction (4. 33) is just one particular reaction obtained from (4. 30). It plays a very important role in tobacco science as well in organic chemistry, because it provides accurate relations among molecules.

By the above organic reaction (4. 33) showed in a molecular form, for the first time in literature the quantitative relations among tobacco alkaloids and toxins are given.

Organic chemistry does not study the quantitative relations among components, and it is one of its biggest shortcomings. By the reactions (4. 33) this gap is filled out.

Now, we shall consider obtained coefficients x_i, $(1 \le i \le 49)$ of the reaction (4.33) as elements of a permutation. Let they represent this permutation $(x_1, x_2, \ldots, x_{49})$, which in fact is obtained from the initial permutation $(x_{18}, x_{19}, x_{17}, x_{37}, x_{36}, x_{35}, x_1, x_{43}, x_{16}, x_{20}, x_{42}, x_9, x_{41}, x_5, x_{40}, x_{15}, x_6, x_{39}, x_7, x_{11}, x_8, x_{12}, x_3, x_{23}, x_{33}, x_{10}, x_4, x_{24}, x_{32}, x_{22}, x_{31}, x_{30}, x_{29}, x_{34}, x_{14}, x_{28}, x_{21}, x_{49}, x_{44}, x_{38}, x_{48}, x_{13}, x_{25}, x_{45}, x_{27}, x_{46}, x_{47}, x_{26}, x_2)$ or arranged permutation of the reaction (4.33) is

$$\sigma = (x_7, x_{49}, x_{23}, x_{27}, x_{14}, x_{17}, x_{19}, x_{21}, x_{12}, x_{26}, x_{20}, x_{22}, x_{42}, x_{35}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_{37}, x_{30}, x_{24}, x_{28}, x_{43}, x_{48}, x_{45}, x_{36}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{40}, x_{18}, x_{15}, x_{13}, x_{11}, x_8, x_{39}, x_{44}, x_{46}, x_{47}, x_{41}, x_{38}).$$

The parity of the above permutation we shall determine on this way.

Since x_7 is before: x_3, x_1, x_2, x_6, x_5, x_4, then we have 6 inversions;

Since x_{49} is before: x_{23}, x_{27}, x_{14}, x_{17}, x_{19}, x_{21}, x_{12}, x_{26}, x_{20}, x_{22}, x_{42}, x_{35}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_{37}, x_{30}, x_{24}, x_{28}, x_{43}, x_{48}, x_{45}, x_{36}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{40}, x_{18}, x_{15}, x_{13}, x_{11}, x_8, x_{39}, x_{44}, x_{46}, x_{47}, x_{41}, x_{38}, then we have 47 inversions;

Since x_{23} is before: x_{14}, x_{17}, x_{19}, x_{21}, x_{12}, x_{20}, x_{22}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 21 inversions;

Since x_{27} is before: x_{14}, x_{17}, x_{19}, x_{21}, x_{12}, x_{26}, x_{20}, x_{22}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_{24}, x_{25}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 24 inversions;

Since x_{14} is before: x_{12}, x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{13}, x_{11}, x_8, then we have 12 inversions;

Since x_{17} is before: x_{12}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{15}, x_{13}, x_{11}, x_8, then we have 14 inversions;

Since x_{19} is before: x_{12}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 15 inversions;

Since x_{21} is before: x_{12}, x_{20}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 16 inversions;

Since x_{12} is before: x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{11}, x_8, then we have 10 inversions;

Since x_{26} is before: x_{20}, x_{22}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_{24}, x_{25}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 18 inversions;

Since x_{20} is before: x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 14 inversions;

Since x_{22} is before: x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 14 inversions;

Since x_{42} is before: x_{35}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_{37}, x_{30}, x_{24}, x_{28}, x_{36}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{40}, x_{18}, x_{15}, x_{13}, x_{11}, x_8, x_{39}, x_{41}, x_{38}, then we have 30 inversions;

Since x_{35} is before: x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_{30}, x_{24}, x_{28}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 23 inversions;

Since x_{16} is before: x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{15}, x_{13}, x_{11}, x_8, then we have 12 inversions;

Since x_9 is before: x_3, x_1, x_2, x_6, x_5, x_4, x_8, then we have 7 inversions;

Since x_3 is before: x_1, x_2, then we have 2 inversions;

Since x_{10} is before: x_6, x_5, x_4, x_8, then we have 4 inversions;

Since x_{37} is before: x_{30}, x_{24}, x_{28}, x_{36}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 18 inversions;

Since x_{30} is before: x_{24}, x_{28}, x_{29}, x_{25}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 12 inversions;

Since x_{24} is before: x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 8 inversions;

Since x_{28} is before: x_{25}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 9 inversions;

Since x_{43} is before: x_{36}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, x_{39}, x_{41}, x_{38}, then we have 18 inversions;

Since x_{48} is before: x_{45}, x_{36}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{40}, x_{18}, x_{15}, x_{13}, x_{11}, x_8, x_{39}, x_{44}, x_{46}, x_{47}, x_{41}, x_{38}, then we have 23 inversions;

Since x_{45} is before: x_{36}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{40}, x_{18}, x_{15}, x_{13}, x_{11}, x_8, x_{39}, x_{44}, x_{41}, x_{38}, then we have 20 inversions;

Since x_{36} is before: x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 14 inversions;

Since x_{33} is before: x_{32}, x_{31}, x_{29}, x_{25}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 12 inversions;

Since x_{32} is before: x_{31}, x_{29}, x_{25}, x_6, x_5, x_4, x_{40}, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 12 inversions;

Since x_{31} is before: x_{29}, x_{25}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 10 inversions;

Since x_{29} is before: x_{25}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 9 inversions;

Since x_{25} is before: x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 8 inversions;

Since x_{34} is before: x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 8 inversions;

Since x_6 is before: x_5, x_4, then we have 2 inversions;

Since x_5 is before: x_4, then we have 1 inversion;

Since x_{40} is before: x_{18}, x_{15}, x_{13}, x_{11}, x_8, x_{39}, x_{38}, then we have 7 inversions;

Since x_{18} is before: x_{15}, x_{13}, x_{11}, x_8, then we have 4 inversions;

Since x_{15} is before: x_{13}, x_{11}, x_8, then we have 3 inversions;

Since x_{13} is before: x_{11}, x_8, then we have 2 inversions;

Since x_{11} is before: x_8, then we have 1 inversion;

Since x_{39} is before: x_{38}, then we have 1 inversion;

Since x_{44} is before: x_{41}, x_{38}, then we have 2 inversions;

Since x_{46} is before: x_{41}, x_{38}, then we have 2 inversions;

Since x_{47} is before: x_{41}, x_{38}, then we have 2 inversions;

Since x_{41} is before: x_{38}, then we have 1 inversion;

The total number of the inversion is k = 498. According to the Definition 3. 37, the sign of the permutation σ will be sign$\sigma = (-1)^k = (-1)^{498} = 1$, and according to the Definition 3. 38, the permutation σ is even.

Elements of the permutation σ of the chemical equation (4. 33), lie in seven orbits.

The orbits of the permutation σ are:

$\mathcal{O}_1 = (x_1, x_2, x_4, x_7, x_8, x_9, x_{12}, x_{15}, x_{16}, x_{18}, x_{19}, x_{21}, x_{22}, x_{25}, x_{27}, x_{29}, x_{30}, x_{32}, x_{33}, x_{37}, x_{38}, x_{39}, x_{40}, x_{43}, x_{44}, x_{45}, x_{49})$ of length $m = 27$,

$\mathcal{O}_2 = (x_3, x_5, x_6, x_{14}, x_{17}, x_{23}, x_{24}, x_{28}, x_{35}, x_{36})$ of length $m = 10$,

$\mathcal{O}_3 = (x_{10}, x_{11}, x_{13}, x_{20}, x_{26}, x_{41}, x_{42}, x_{48})$ of length $m = 8$,

$\mathcal{O}_4 = (x_{31})$ of length $m = 1$,

$\mathcal{O}_5 = (x_{34})$ of length $m = 1$,

$\mathcal{O}_6 = (x_{46})$ of length $m = 1$

and

$\mathcal{O}_7 = (x_{47})$ of length $m = 1$.

Therefore, the permutation can be presented in this form

$$\sigma = \mathcal{O}_1 \cup \mathcal{O}_2 \cup \mathcal{O}_3 \cup \mathcal{O}_4 \cup \mathcal{O}_5 \cup \mathcal{O}_6 \cup \mathcal{O}_7.$$

One of the most important criteria for balancing chemical equations is their stability. Stability made analysis shown that this reaction (4. 33) is not stable.

In the next section we shall consider the field temperature problem in the combustion zone by using of two-dimensional heat equation.

5. Field Temperature Problem

Consider the two-dimensional heat equation on the rectangular region $0 < x < \ell$, $0 < y < h$,

$$\partial T/\partial t = k\,(\partial^2 T/\partial^2 x + (\partial^2 T/\partial^2 y), \quad (5.\ 1)$$

with the initial condition

$$T(x, y, 0) = f(x, y), \quad (5.\ 2)$$

and these boundary conditions

$$T(0, y, t) = 0,\ \partial T(\ell, y, t)/\partial x = 0, \quad (5.\ 3)$$

$$\partial T(x, 0, t)/\partial y = 0,\ \partial T(x, h, t)/\partial y = 0,\ t \to \infty.$$

Since the equation and the boundary conditions are linear and homogeneous we can use separation of variables. Assuming a solution of the form $T(x, y, t) = X(x)\cdot Y(y)\cdot T(t)$ and substituting this into the partial differential equation (5. 1) we have

$$T'(t)X(x)Y(y)$$
$$= k\,[X''(x)Y(y)T(t) + Y''(y)X(x)T(t)],$$

so that

$$T'(t)/kT(t) = X''(x)/X(x) + Y''(y)/Y(y) = -\lambda,$$

where λ is the separation constant. This gives

$$X''(x)/X(x) = -\lambda - Y''(y)/Y(y) = -\mu,$$

where μ is another separation constant.

We can satisfy the boundary conditions by requiring that

$$X(0) = X'(\ell) = 0 \text{ and } Y'(0) = Y'(h) = 0,$$

and therefore X and Y satisfy these boundary value problems

$$X''(x) + \mu X(x) = 0,\ 0 < x < \ell, \quad (5.\ 4)$$
$$X(0) = 0 \text{ and } X'(\ell) = 0,$$

and

$$Y''(y) + \alpha Y(y) = 0,\ 0 < y < h, \quad (5.\ 5)$$
$$Y'(0) = 0 \text{ and } Y'(h) = 0,$$

where $\alpha = \lambda - \mu$, while T satisfies the differential equation

$$T' + \lambda kT = 0,\ t > 0. \quad (5.\ 6)$$

The eigenvalues and corresponding eigenfunctions for the problem (5. 4) are

$$\mu_n = [(2n - 1)\pi/2\ell]^2$$

and

$$X_n(x) = \sin[(2n - 1)\pi x/2\ell],\ n = 1, 2, 3, \ldots$$

while the eigenvalues and corresponding eigenfunctions for the problem (5. 5) are

$$\alpha_m = (m\pi/h)^2$$

and

$$Y_m(y) = \cos(m\pi y/h),\ m = 0, 1, 2, \ldots$$

The corresponding solutions to the time equation (5. 6) are

$$T_{nm} = \exp(-\lambda_{nm} kt),$$

where

$$\lambda_{nm} = \mu_n + \alpha_m = [(2n - 1)\pi/2\ell]^2 + (m\pi/h)^2,$$

we need to use all possible combinations of indices $n = 1, 2, 3, \ldots$ and $m = 0, 1, 2, \ldots$

The products

$$T_{nm}(x, y, t) = X_n(x)\cdot Y_m(y)\cdot T_{nm}(t)$$
$$= \sin[(2n - 1)\pi x/2\ell]\cdot\cos(m\pi y/h)\cdot\exp(-\lambda_{nm} kt),$$

satisfy the partial differential equation and the boundary conditions, and by the superposition principle, the function

$$T(x, y, t) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} C_{nm} \sin[(2n - 1)\pi x/2\ell]$$
$$\times\cos(m\pi y/h)\exp(-\lambda_{nm} kt), \quad (5.\ 7)$$

also satisfies the partial differential equation and all the boundary conditions. In order to satisfy the initial condition, we could use the fact that the eigenfunctions

$$\{\sin[(2n - 1)\pi x/2\ell]\cdot\cos(m\pi y/h)\}_{n\geq 1,\ m\geq 0}$$

form an orthogonal set on the rectangle $[0, \ell]\times[0,h]$ in $\mathbb{R}^2$. However, we use another method which is similar to the methods used for one-dimensional Fourier series expansions. Setting

$t = 0$ in the expression above for $T(x, y, t)$, we want

$$18 = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} C_{nm} \sin[(2n - 1)\pi x/2\ell]\cdot\cos(m\pi y/h),$$

for $0 \le x \le \ell$ and $0 \le y \le h$, and writing this as

$$18 = \sum_{n=1}^{\infty} B_n(y)\sin[(2n - 1)\pi x/2\ell].$$

This is a Fourier sine series expansion of 18 on the interval $[0,\ell]$ holding y fixed, and therefore

$B_n(y) = (36/\ell)\ {}_0\int^{\ell} \sin[(2n - 1)\pi x/2\ell]dx$, $(n \ge 1)$.

For $n \ge 1$,

$$B_n(y) = \sum_{m=0}^{\infty} C_{nm} \cos(m\pi y/h),$$

is the expansion of $B_n(y)$ on the interval $[0,h]$, so that

$$C_{n0} = (1/h)\ {}_0\int^{h} B_n(y)dy$$

$$= (36/\ell h)\ {}_0\int^{h}\ {}_0\int^{\ell} \sin[(2n - 1)\pi x/2\ell]dxdy,\ (n \ge 1).$$

$$C_{nm} = (2/h)\ {}_0\int^{h} B_n(y)\cos(m\pi y/h)dy$$

$$= (72/\ell h)\ {}_0\int^{h}\ {}_0\int^{\ell} \sin[(2n - 1)\pi x/2\ell] \times\cos(m\pi y/h)dxdy,\ (\forall\ n, m \ge 1).$$

We can note that that in the solution (5. 7) all terms in the sum for which either $n \ge 1$ or $m \ge 1$ contain a factor of $\exp(-\lambda_{nm}kt)$, where $\lambda_{nm} > 0$ and as $t \to \infty$, all these terms vanish, and therefore $\lim_{t\to\infty} T(x, y, t) = 0$.

According to the above obtained results, on the Fig. 3 are presented the particular $\mathbb{R}^2$ isotherms for $T(x, y, 0) = 18$ºC, $0 < x < 8$ mm, $0 < y < 10$ mm, and $k = 1\cdot10^{-7}$ m^2/s in the combustion zone for the free burn profile.

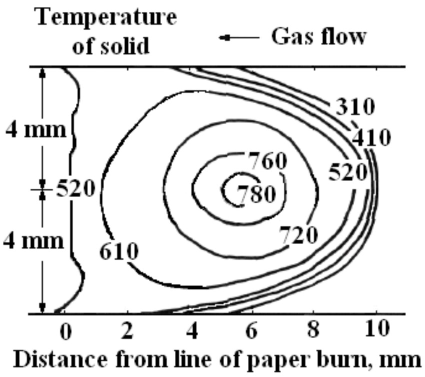

Fig. 3. *Temperature distribution in* $\mathbb{R}^2$, ºC

6. Smoke Filtration Problem

Let us consider a regular cigarette being smoked under the following conditions:

1° In the cigarette smoking process under continuous inhalation (*i. e.*, with a constant speed of the air and burning gases passing through the cigarette and a constant combustion factor) there enters into the cigarette a constant fraction α of any components X_i, $(1 \le i \le n)$ of tobacco (volatiles oils, carbohydrates, proteins, nicotine, organic acids, and so on.), while the remaining fraction $1 - \alpha$ escapes into atmosphere.

2° In filtration of any components X_i, $(1 \le i \le n)$ of tobacco through the cigarette, the absorption factor of the tobacco β is constant.

3° We ignore the change in length of the cigarette during the time of passage of smoke through it.

Remark. *The coefficients* α *and* β *can be naturally distinguished for different components* X_i, $(1 \le i \le n)$ *of tobacco*.

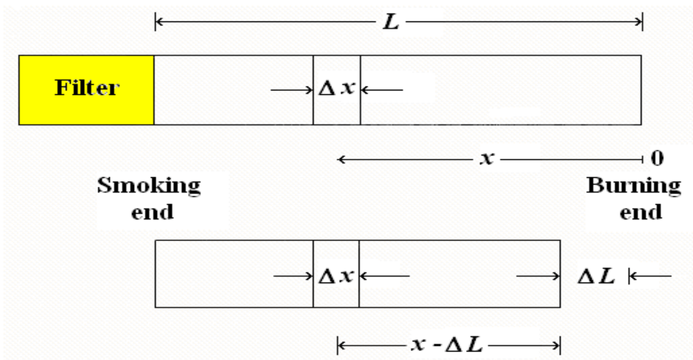

Fig. 4. *A schematic view of a cigarette*

Let $C_i(x, L)$ denote the concentration of components X_i, $(1 \le i \le n)$ for a cigarette of length L at a distance x from the burning end as it is showed in Fig. 4.

The amount of the components X_i, $(1 \le i \le n)$ transmitted down the cigarette after burning ΔL of cigarette is, to first order terms, $\Delta S =$

$\alpha C_i(0, L)\Delta L$. The fraction of ΔS filtered out by element Δx is

$$\Delta(\Delta S) = \Delta C_i \cdot \Delta x = \alpha\beta e^{-\beta x} C_i(0, L)\Delta L \cdot \Delta x. \quad (6.\ 1)$$

This follows since the filtration law is

$$dS/dx = -\beta S \text{ or } S = S_0 e^{-\beta x},$$

and thus, to first order terms, $\Delta S = \beta S \Delta x$.

Equation (6. 1) can be rewritten as

$$[C_i(x - \Delta L, L - \Delta L) - C_i(x, L)]/\Delta L = \alpha\beta C_i(0, L)e^{-\beta x}.$$

On letting $\Delta L \to 0$, one obtains the mixed partial differential equation of first order

$$\partial C_i(x, L)/\partial x + \partial C_i(x, L)/\partial L = -\alpha\beta C_i(0, L)e^{-\beta x}, \quad (6.\ 2)$$

subject to the initial condition $C_i(0, L_0) = C_0$ (constant), where L_0 is the initial length of the cigarette.

To solve the above partial equation (6. 2) we suppose a solution of the form

$$C_i(x, L) = C_0 + \varphi(L)e^{-\beta x},$$

where $\varphi(L_0) = 0$. This transforms partial equation (6. 2) into following ordinary linear differential equation

$$\varphi'(L) - \beta(1 - \alpha)\varphi(L) = -\alpha\beta C_0.$$

Therefore

$$C_i(x, L) = C_0 + C_0\alpha e^{-\beta x}[1 - e^{-\beta(1 - \alpha)(L_0 - L)}]/(1 - \alpha).$$

The amount of components X_i, $(1 \le i \le n)$ transmitted to the smoker after burning ΔL of the cigarette is, to first order terms,

$$\Delta S_t = \alpha C_i(0, L)e^{-\beta L}\Delta L.$$

Therefore, the total amount transmitted when the cigarette has been burned down to a length L_f is

$$S_t = \int_{L_f}^{L_0} \alpha C_i(0, L)e^{-\beta L}dL \quad (6.\ 3)$$
$$= \alpha C_0 e^{-\beta L_f}[1 - e^{-\beta(1 - \alpha)(L_0 - L_f)}]/(1 - \alpha)\beta$$
$$= [C_i(L_f, L_f) - C_0]/\beta.$$

The total amount of X_i, $(1 \le i \le n)$ destroyed by burning is

$$S_b = \int_{L_f}^{L_0} (1 - \alpha)C_i(0, L)Dl = C_0(L_0 - L_f) - \alpha C_0[1 - e^{-\beta(1 - \alpha)(L_0 - L_f)}]/(1 - \alpha)\beta.$$

The amount of X_i, $(1 \le i \le n)$ left in the cigarette is

$$S_c = \int_0^{L_f} C_i(x, L_f)dx = C_0 L_f + \alpha C_0(1 - e^{-\beta L_f})[1 - e^{-\beta(1 - \alpha)(L_0 - L_f)}]/(1 - \alpha)\beta.$$

It follows immediately that

$$S_t + S_b + S_c = C_0 L_0. \quad (6.\ 4)$$

Equation (4) expresses the material balance on components X_i, $(1 \le i \le n)$.

We can now determine the effect of the initial length L_0 on the amount S_t transmitted to the smoker. Let as consider two cigarettes of lengths L_0 and L_0', and identical otherwise. The amounts S_t and S_t' may be compared on one of the following two principles:

(P_1) both cigarettes burn down the same amount,

(P_2) both cigarettes burn down to the same final length.

In the latter case, we would consider the amount S_t per unite length of cigarette smoked. On either principle, the longer cigarette is more effective in reducing the amount S_t transmitted to the smoker.

We shall now consider the effects of filtration alone on the amount S_t. We let $\alpha \to 1$ (zero burning fraction). Then (6. 3) reduces to

$$S_t = C_0 e^{-\beta L_f}(L_0 - L_f).$$

Using principle (P_1) for a comparison we get

$$S_t/S_t' = e^{-\beta L_f}/e^{-\beta L_f'}. \quad (6.\ 5)$$

Therefore, the initially longer cigarette is more effective in reducing the amount of S_t transmitted. Though, if we now use principle (P_2) we obtain

$$S_t/(L_0 - L_f) = S_t'/(L_0' - L_f). \quad (6.\ 6)$$

In this case, the filtering capacity is independent of the initial length. Since many smokers burn their cigarettes (be they regular or king size) to the same approximate final length before discarding them, principle (P_2) would be indicated.

To approximate more closely to actual smoking conditions, we shall consider a cigarette being smoked in the following way. The cigarette is first smoked a length ΔL_s under steady inhalation and then is allowed to burn a length ΔL_b without inhalation. This process is repeated until the cigarette is discarded.

We now have to solve the partial differential equation (6. 2) subject to the nonconstant initial condition $C_i(x, L_0) = C(x)$, where $C(x)$ is a special function. To solve (6. 2) under this condition, we let

$$C_i(x, L) = e^{-\beta x}[U(x, L) + V(L)].$$

Then (6. 2) becomes

$$\partial U(x, L)/\partial x + \partial U(x, L)/\partial L + V'(L) = \beta[U(x, L) + V(L)] - \alpha\beta[U(0, L) + V(L)].$$

If we let

$$\partial U(x, L)/\partial x + \partial U(x, L)/\partial L = \beta U(x, L), \quad (6.\ 7)$$

subject to the condition

$$U(x, L_0) = e^{\beta x}C(x), \quad (6.\ 8)$$

then $V(L)$ must satisfy

$$dV(L)/dL - \beta(1 - \alpha)V(L) = -\alpha\beta U(0, L), \quad (6.\ 9)$$

subject to the initial condition $V(L_0) = 0$.

The subsidiary equations for solving partial differential equation (7) are

$$dx = dL = dU(x, L)/\beta.$$

Consequently,

$$U(x, L) = e^{\beta x}\zeta(L - x),$$

where $\zeta(x)$ is an arbitrary function of x. To determine $\zeta(x)$, we use (6. 8). Thus we find that

$$U(x, L) = e^{\beta x}C(L_0 - L + x).$$

We can now solve (6. 9), and the solution is

$$V(L) = \alpha\beta e^{\beta(1 - \alpha)L} {}_L\int^{L_0} e^{-\beta(1 - \alpha)L}C(L_0 - L)dL,$$

whence

$$C(x, L) = C(L_0 - L + x)$$
$$+ \alpha\beta e^{\beta(1 - \alpha)L - \beta x} {}_L\int^{L_0} e^{-\beta(1 - \alpha)L}C(L_0 - L)dL.$$

If we now smoke a length ΔL_s under steady inhalation, the concentration function will be given by

$$C_1(x, L) = C_0$$
$$+ \alpha\beta e^{-\beta x + \beta(1 - \alpha)L_1} {}_{L_1}\int^{L_0} e^{-\beta(1 - \alpha)L}C_0dL,$$

where $L_1 = L_0 - \Delta L_s$. After burning a length ΔL_b, concentration function will then be

$$C_1^*(x, L_2) = C_1(x + \Delta L_b, L_1).$$

Here we have

$$L_2 = L_1 - \Delta L_b = L_0 - \Delta L_s - \Delta L_b.$$

On repeating another cycle we obtain

$$C_2(x, L_3) = C_1^*(L_2 - L_3 + x, L_2)$$
$$+ \alpha\beta e^{-\beta x + \beta(1 - \alpha)L_3}$$
$$\times {}_{L_3}\int^{L_2} e^{-\beta(1 - \alpha)L}C_1^*(L_2 - L, L_2)dL,$$
$$C_2^*(x, L_4) = C_2(x + \Delta L_b, L_3).$$

By induction, we find that after n cycles,

$$C_n(x, L_{2n-1}) = C_{n-1}^*(L_{2n-2} - L_{2n-1} + x, L_{2n-2})$$
$$+ \alpha\beta e^{-\beta x + \beta(1 - \alpha)L_{2n-1}}$$
$$\times {}_{L_{2n-1}}\int^{L_{2n-2}} e^{-\beta(1 - \alpha)L}C_{n-1}^*(L_{2n-2} - L, L_{2n-2})dL,$$
$$C_n^*(x, L_{2n}) = C_n(x + \Delta L_b, L_{2n-1}),$$

where

$$L_{2n-1} = L_0 - n\Delta L_s - (n - 1)\Delta L_b,$$
$$L_{2n} = L_0 - n\Delta L_s - n\Delta L_b.$$

The amount S_{tn} transmitted to the smoker during the n-th cycle is given by

$$S_{tn} = {}_{L_{2n-1}}\int^{L_{2n-2}} \alpha C_{n-1}(0, L)e^{-\beta L}dL.$$

Thus, the total amount S_T absorbed by the smoker after n cycles is

$$S_T = \sum_{n=1}^{N} S_{tn}$$

$$= \sum_{n=1}^{N} {}_{L_{2n-1}}\int^{L_{2n-2}} \alpha C_{n-1}(0, L)e^{-\beta L}dL.$$

Again, in order to determine the effect of filtration alone, we set $\alpha = 1$. It then can be established by induction that

$$C_N(x, L)/C_0 = 1$$
$$+ \beta\Delta L_s e^{-\beta x}[(e^{-(N - 1)\beta\Delta L_b} - 1)/(1 - e^{\beta\Delta L_b})$$
$$+ (L_0 - L)/\Delta L_s - (N - 1)(1 + \Delta L_b/\Delta L_s)],$$

and that also

$$S_T = C_0\Delta L_s e^{\beta(N\Delta L_s - L_0)}\times[(1 - e^{N\beta\Delta L_b})/(1 - e^{\beta\Delta L_b})].$$

For a comparison of the filtration efficiency of two cigarettes (under discontinuous inhalation), identical except for length, it is reasonable to keep ΔL_s and ΔL_b fixed. Then, using principle (P_1) we get

$$S_T / S_T' = e^{-\beta L_0}/e^{-\beta L_0'},$$

which corresponds to (6. 5), and to which it reduces if $\Delta L_b = 0$. By using principle (P_2) we obtain

$$(S_T/N\Delta L_s)/(S_T'/N'\Delta L_s)$$
$$= [(1 - e^{-N\beta\Delta L_b})/N]/[(1 - e^{-N'\beta\Delta L_b})/N'].$$

Since $(1 - e^{-x})/x$ is a monotonic decreasing function, it follows that under either principles, (P_1) and (P_2), the longer cigarette is a better filter under discontinuous inhalation which more nearly approximates actual smoking conditions than continuous inhalation.

7. GROUPS' FORMATION PROBLEM

I. *A symmetric group* S_{49}

Let G be the symmetric group S_{49}. We have $|G|$ = 608281864034267560872252163321295376887552831379210240000000000 $= 2^{46}\times3^{22}\times5^{10}\times7^8\times11^4\times13^3\times17^2\times19^2\times23^2\times29\times31\times37\times41\times43\times47$.

Generators of G:	a_1 = (1,2,…,49) (order 49)
	a_2 = (1,2) (order 2)

The center of G is trivial. The derived subgroup $D = [G, G]$ is a simple group of order 304140932017133780436126081660647688443776415689605120000000000, generated by {(1,8,42,39,49,46,9,24,12,40,23,21,16,44,37,6,7,11,22,4,34,19)(2,36,33,18,47,20,28,10,26,3,25,35,17,32,27)(5,15,31,43,48,14,30,41,13,45)(1,20,18,43,17,25,4,40,45,42,24,36,46,3,22,27,41,13,14,31,44,32,7,33,47)(2,12,35,49,28,21,9,30,16,39,6,34,38,23,8,10,37,19,29,5,26,11,48)} and $G/D \cong C_2$.

Let G be a group generated by 2 permutations. We have

$|G|$ = 304140932017133780436126081660647688443776415689605120000000000 $= 2^{45}\times3^{22}\times5^{10}\times7^8\times11^4\times13^3\times17^2\times19^2\times23^2\times29\times31\times37\times41\times43\times47$.

G is the alternating group A_{49} on the set {1, 2, …, 49}. It's a simple group.

II. *An alternating group* A_{49}

Let *G* be the alternating group A_{49}. We have

$|G|$ = 30414093201713378043612608166064768844377641568960512000000000000

$= 2^{45}\times3^{22}\times5^{10}\times7^{8}\times11^{4}\times13^{3}\times17^{2}\times19^{2}\times23^{2}\times29\times31\times37\times41\times43\times47$.

Generators of *G*:	a_1 = (1,2,… ,49) (order 49)
	a_2 = (47,48,49) (order 3)

III. *A primitive group of degree* 49

There are in total 40 primitive groups (up to conjugation) on the set {1, 2, …, 49}. Only 38 of them are listed below, the rest being too big (alternating or symmetric).

Number	Order	Nature
1	196	solvable
2	294	solvable
3	392	solvable
4	392	solvable
5	392	solvable
6	588	solvable
7	588	solvable
8	588	solvable
9	784	solvable
10	784	solvable
11	784	solvable
12	882	solvable
13	1176	solvable
14	1176	solvable
15	1176	solvable
16	1176	solvable
17	1176	solvable
18	1176	solvable
19	1568	solvable
20	1764	solvable
21	1764	solvable
22	2352	solvable
23	2352	solvable
24	2352	solvable
25	2352	solvable
26	3528	solvable
27	3528	solvable
28	4704	solvable
29	7056	solvable
30	49	cyclic
31	147	solvable
32	294	solvable
33	98	solvable
34	56448	---
35	12700800	---
36	25401600	---
37	25401600	---
38	50803200	---

1. Let *G* be a primitive group of degree 49 with 2 generators. We have $|G| = 196 = 2^2\times7^2$.

Generators of *G*:	a_1 = (2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25) (order 4)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

G is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup $D = [G, G]$ is an abelian group of order 49, generated by {(1,16,32,24,48,14,40)(2,18,31,27,49,12,36)(3,20,35,26,43,9,39)(4,17,34,28,47,8,37)(5,15,30,25,45,13,42)(6,21,33,22,44,11,38)(7,19,29,23,46,10,41)(1,26,42,34,10,18,44)(2,22,40,35,13,17,46)(3,25,37,29,12,21,48)(4,23,36,33,14,20,45)(5,28,41,31,11,16,43)(6,24,39,30,8,19,49)(7,27,38,32,9,15,47)} and $G/D \cong C_4$.

Lower central series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	C_4

Derived series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	C_4
2	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
3	4	cyclic	49	2	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)
4	7	cyclic	2	1	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
5	7	cyclic	2	1	(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)
6	7	cyclic	2	1	(1,10,26,18,42,44,34)(2,13,22,17,40,46,35)(3,12,25,21,37,48,29)(4,14,23,20,36,45,33)(5,11,28,16,41,43,31)(6,8,24,19,39,49,30)(7,9,27,15,38,47,32)
7	7	cyclic	2	1	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
8	14	dihedral	14	2,7	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
9	14	dihedral	14	2,4	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
10	14	dihedral	14	2,5	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,

					14,26,15,37,46,31)(7,12,22,16,39,45,34)
11	14	dihedral	14	2,6	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46),(1,10,26,18,42,44,34)(2,13,22,17,40,46,35)(3,12,25,21,37,48,29)(4,14,23,20,36,45,33)(5,11,28,16,41,43,31)(6,8,24,19,39,49,30)(7,9,27,15,38,47,32)
12	49	abelian	1	4,5,6,7	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
13	98	solvable	1	8,9,10,11,12	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
14	196	*G*	1	3,13	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25),(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	cyclic	--	(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31), (1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
2	98	solvable	cyclic	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40),(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	4	49	cyclic	4	*G*	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29, 19)(9,13,33,31)(10,24,34,48)(11,43,35,22) (14,18,32,42)(15,45,36,27)(16,26,40,44) (17,21,41,39)(23,49,47,25)
7	49	1	abelian	*G*	49	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37) (3,17,31,24,45,10,38)(4,18,32,25,46,11,39) (5,19,33,26,47,12,40)(6,20,34,27,48,13,41) (7,21,35,28,49,14,42),(1,45,19,11,35,37,27) (2,48,15,10,33,39,28)(3,47,18,14,30,41,22) (4,49,16,13,29,38,26)(5,46,21,9,34,36,24) (6,43,17,12,32,42,23)(7,44,20,8,31,40,25)

2. Let *G* be a primitive group of degree 49 with 3 generators. We have $|G| = 294 = 2\times3\times7^2$.

Generators of *G*:	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30) (13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39) (28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42) (13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48) (19,31,49)(20,32,44)(21,33,45) (order 3)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31) (4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. *G* is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup *D* = [*G*, *G*] is a solvable group of order 147, generated by {(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,30,46,38,20,28,12)(2,32,45,41, 21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7, 33,43,37,18,24,13)} and $G/D \cong C_2$.

Lower central series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	C_2

Derived series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	C_2
2	49	abelian	dihedral	C_3
3	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	21	1	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16) (11,23)(12,30)(13,37)(14,44)(18,24)(19,31) (20,38)(21,45)(26,32)(27,39)(28,46)

					(34,40)(35,47)(42,48)
3	3	cyclic	49	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
4	6	dihedral	49	2,3	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
5	7	cyclic	2	1	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
6	7	cyclic	3	1	(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)
7	7	cyclic	3	1	(1,10,26,18,42,44,34)(2,13,22,17,40,46,35)(3,12,25,21,37,48,29)(4,14,23,20,36,45,33)(5,11,28,16,41,43,31)(6,8,24,19,39,49,30)(7,9,27,15,38,47,32)
8	14	dihedral	3	2,7	(2,22)(3,29)(4,36)(5,43)(6,8)(7,15)(9,27)(10,34)(11,41)(12,48)(14,20)(16,28)(17,35)(18,42)(19,49)(24,30)(25,37)(26,44)(32,38)(33,45)(40,46), (1,10,26,18,42,44,34)(2,13,22,17,40,46,35)(3,12,25,21,37,48,29)(4,14,23,20,36,45,33)(5,11,28,16,41,43,31)(6,8,24,19,39,49,30)(7,9,27,15,38,47,32)
9	14	cyclic	21	2,6	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48),(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)
10	21	solvable	14	3,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
11	49	abelian	1	5,6,7	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

12	98	solvable	3	8,9,11	(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13), (1,41,9,49,25,33,17)(2,48,11,35,24,5,20,8,42,23,47,18,29,38)(3,6,13,14,28,26,19,15,36,37,44,46,32,31)(4,34,10,7,27,12,21,22,40,16,43,39,30,45)
13	147	solvable	1	10,11	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)
14	294	*G*	1	4,12,13	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,7,49,45,17,16,9,12,33,34,41,39,25,22)(2,14,47,31,20,37,11,26,29,6,42,46,24,15)(3,21,44,10,19,30,13,40,32,27,36,4,28,43)(5,35,48,38,18,23,8)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	147	solvable	cyclic	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	2	21	cyclic	14	*G*	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
3	3	49	cyclic	6	147	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
7	49	1	abelian	*G*	49	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46),(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)

3. Let *G* be a primitive group of degree 49 with 4 generators. We have $|G| = 392 = 2^3 \times 7^2$.

Generators of G:	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49) (order 2)
	a_3 = (2,5)(3,6)(4,7)(9,12)(10,13)(11,14)(16,19)(17,20)(18,21)(23,26)(24,27)(25,28)(30,33)(31,34)(32,35)(37,40)(38,41)(39,42)(44,47)(45,48)(46,49) (order 2)
	a_4 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 98, generated by {(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,45,19,11,35,37,27)(2,48,15,10,33,39,28)(3,47,18,14,30,41,22)(4,49,16,13,29,38,26)(5,46,21,9,34,36,24)(6,43,17,12,32,42,3)(7,44,20,8,31,40,25)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)} and $G/D \cong C_2^{\,2}$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	$C_2^{\,2}$
2	49	abelian	nilpotent	C_2

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	$C_2^{\,2}$
2	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	$C_2^{\,2}$
2	49	abelian	nilpotent	C_2
3	1	trivial	solvable	$C_7^{\,2}$

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	14	1	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
3	2	cyclic	14	1	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
					(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,

4	2	cyclic	49	1	40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
5	4	abelian	49	3,4	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
6	4	cyclic	49	4	(2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46)
7	4	abelian	49	2,4	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(2,5)(3,6)(4,7)(9,12)(10,13)(11,14)(16,19)(17,20)(18,21)(23,26)(24,27)(25,28)(30,33)(31,34)(32,35)(37,40)(38,41)(39,42)(44,47)(45,48)(46,49)
8	7	cyclic	2	1	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
9	7	cyclic	2	1	(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)
10	7	yclic	4	1	(1,10,26,18,42,44,34)(2,13,22,17,40,46,35)(3,12,25,21,37,48,29)(4,14,23,20,36,45,33)(5,11,28,16,41,43,31)(6,8,24,19,39,49,30)(7,9,27,15,38,47,32)
11	8	nilpotent	49	5,6,7	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
12	14	dihedral	2	2,8	(2,5)(3,6)(4,7)(9,12)(10,13)(11,14)(16,19)(17,20)(18,21)(23,26)(24,27)(25,28)(30,33)(31,34)(32,35)(37,40)(38,41)(39,42)(44,47)(45,48)(46,49)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
					(2,29)(3,36)(4,43)(5,8)(6,15)(7,22)(9,33)(10,40)(11,47)(13,19)(14,26)(16,34)(17,41)(18,48)(21,27)(23,35)(24,

13	14	dihedral	2	3,9	42)(25,49)(31,37)(32,44)(39,45)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)
14	14	cyclic	14	3,9	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)
15	14	dihedral	14	4,8	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
16	14	dihedral	14	4,9	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)
17	14	cyclic	14	2,8	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
18	14	dihedral	28	4,10	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46),(1,10,26,18,42,44,34)(2,13,22,17,40,46,35)(3,12,25,21,37,48,29)(4,14,23,20,36,45,33)(5,11,28,16,41,43,31)(6,8,24,19,39,49,30)(7,9,27,15,38,47,32)
19	28	dihedral	14	7,12,15,17	(2,5)(3,6)(4,7)(9,12)(10,13)(11,14)(16,19)(17,20)(18,21)(23,26)(24,27)(25,28)(30,33)(31,34)(32,35)(37,40)(38,41)(39,42)(44,47)(45,48)(46,49)(1,7,3,2,5,6,4)(8,35,10,30,12,34,11,29,

					14,31,9,33,13,32)(15,42,17,37,19,41, 18,36,21,38,16,40,20,39)(22,49,24,44, 26,48,25,43,28,45,23,47,27,46)
20	28	dihedral	14	5,13,14,16	(1,9)(3,44)(4,30)(5,23)(6,37)(7,16)(10, 43)(11,29)(12,22)(13,36)(14,15)(17, 49)(18,35)(19,28)(20,42)(24,47)(25, 33)(27,40)(31,46)(34,39)(38,48)(1,49, 17,9,33,41,25)(2,35,20,23,29,48,24,8, 47,38,11,5,42,18)(3,14,19,37,32,6,28, 15,44,31,13,26,36,46)(4,7,21,16,30,34, 27,22,43,45,10,12,40,39)
21	49	abelian	1	8,9,10	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15, 16,18,17,20,21,19)(22,23,25,24,27,28, 26)(29,30,32,31,34,35,33)(36,37,39, 38,41,42,40)(43,44,46,45,48,49,47)(1, 8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46, 32)(5,12,26,19,40,47,33)(6,13,27,20,41, 48,34)(7,14,28,21,42,49,35)
22	98	solvable	1	15,16,18,21	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11, 35)(12,30)(13,31)(14,32)(15,36)(16, 40)(17,41)(18,42)(19,37)(20,38)(21, 39)(22,43)(23,47)(24,48)(25,49)(26, 44)(27,45)(28,46)(1,8,22,15,36,43,29) (2,9,23,16,37,44,30)(3,10,24,17,38,45, 31)(4,11,25,18,39,46,32)(5,12,26,19, 40,47,33)(6,13,27,20,41,48,34)(7,14, 28,21,42,49,35)(1,2,4,3,6,7,5)(8,9,11, 10,13,14,12)(15,16,18,17,20,21,19) (22,23,25,24,27,28,26)(29,30,32,31, 34,35,33)(36,37,39,38,41,42,40)(43, 44,46,45,48,49,47)
23	98	solvable	2	12,17,21	(1,8,22,15,36,43,29)(2,12,23,19,37,47, 30,5,9,26,16,40,44,33)(3,13,24,20,38, 48,31,6,10,27,17,41,45,34)(4,14,25,21, 39,49,32,7,11,28,18,42,46,35)(1,7,3,2, 5,6,4)(8,14,10,9,12,13,11)(15,21,17,16, 19,20,18)(22,28,24,23,26,27,25)(29,35, 31,30,33,34,32)(36,42,38,37,40,41,39) (43,49,45,44,47,48,46)
24	98	solvable	2	13,14,21	(1,30,46,38,20,28,12)(2,32,45,41,21, 26,8)(3,34,49,40,15,23,11)(4,31,48, 42,19,22,9)(5,29,44,39,17,27,14)(6, 35,47,36,16,25,10)(7,33,43,37,18,24, 13),(1,41,9,49,25,33,17)(2,48,11,35, 24,5,20,8,42,23,47,18,29,38)(3,6,13, 14,28,26,19,15,36,37,44,46,32,31)(4, 34,10,7,27,12,21,22,40,16,43,39,30,45)
25	196	solvable	1	19,22,23	(1,7,3,2,5,6,4)(8,35,10,30,12,34,11, 29,14,31,9,33,13,32)(15,42,17,37, 19,41,18,36,21,38,16,40,20,39)(22, 49,24,44,26,48,25,43,28,45,23,47,27, 46)(1,43,15,8,29,36,22)(2,47,16,12, 30,40,23,5,44,19,9,33,37,26)(3,48,17,

					13,31,41,24,6,45,20,10,34,38,27)(4,49,18,14,32,42,25,7,46,21,11,35,39,28)
26	196	solvable	1	20,22,24	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,8,9,23,25,18,17,38,41,48,49,35,33,5)(2,22,11,16,24,39,20,45,42,34,47,7,29,12)(3,36,13,44,28,32,19)(4,15,10,37,27,46,21,31,40,6,43,14,30,26)
27	196	solvable	1	6,22	(2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
28	392	*G*	1	11,25,26,27	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(1,15,17,31,33,26,25,46,49,14,9,37,41,6)(2,36,20,3,29,19,24,32,47,28,11,44,42,13)(4,43,21,10,30,40,27)(5,22,18,45,35,12,23,39,48,7,8,16,38,34)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	196	solvable	cyclic	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28),(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)(7,35,49,42,21,28,14)
2	196	solvable	cyclic	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46),(1,9,4,10,6,14,5,8,2,11,3,13,7,12)(15,44,18,45,20,49,19,43,16,46,17,48,21,47)(22,30,25,31,27,35,26,29,23,32,24,34,28,33)(36,37,39,38,41,42,40)
3	196	solvable	cyclic	(1,49,17,9,33,41,25)(2,35,20,23,29,48,24,8,47,38,11,5,42,18)(3,14,19,37,32,6,28,15,44,31,13,26,36,46)(4,7,21,16,30,34,27,22,43,45,10,12,40,39)(1,28,38,30,12,20,46)(2,14,41,44,8,27,45,29,26,17,32,5,21,39)(3,35,40,16,11,6,49,36,23,10,34,47,15,25)(4,7,42,37,9,13,48)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	nilpotent	8	*G*	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
7	49	1	abelian	*G*	49	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42),(1,41,9,49,25,33,17)(2,42,11,47,24,29,20)(3,36,13,44,28,32,19)(4,40,10,43,27,30,21)(5,38,8,48,23,35,18)(6,37,14,46,26,31,15)(7,39,12,45,22,34,16)

4. Let *G* be a primitive group of degree 49 with 3 generators. We have $|G| = 392 = 2^3 \times 7^2$.

Generators of *G*:	a_1 = (2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45) (order 4)
	a_2 = (2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25) (order 4)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. *G* is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup *D* = [*G*, *G*] is a solvable group of order 98, generated by {(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,13,23,21,39,47,31)(2,14,25,19,38,43,34)(3,8,27,16,42,46,33)(4,12,24,15,41,44,35)(5,10,22,20,37,49,32)(6,9,28,18,40,45,29)(7,11,26,17,36,48,30)(1,24,40,32,14,16,48)(2,27,36,31,12,18,49)(3,26,39,35,9,20,43)(4,28,37,34,8,17,47)(5,25,42,30,13,15,45)(6,22,38,33,11,21,44)(7,23,41,29,10,19,46)} and $G/D \cong C_2^2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	C_2^2
2	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	$C_2^{\ 2}$
2	49	abelian	nilpotent	C_2
3	1	trivial	solvable	$C_7^{\ 2}$

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	-
2	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18, 42)(19,37)(20,38)(21,39)(22,43)(23,47)(24, 48)(25,49)(26,44)(27,45)(28,46)
3	4	cyclic	49	2	(2,24,5,48)(3,32,6,14)(4,40,7,16)(8,39,29,21)(9,35,33,11)(10,20,34,38)(12,26,30,44)(13,43, 31,22)(15,47,36,23)(17,37,41,19)(18,28,42, 46)(25,45,49,27)
4	4	cyclic	49	2	(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,16,30,40)(14,28, 32,46)(15,49,36,25)(19,39,37,21)(20,24,38, 48)(23,27,47,45)
5	4	cyclic	49	2	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42, 32,18)(15,27,36,45)(16,44,40,26)(17,39,41, 21)(23,25,47,49)
6	7	cyclic	4	1	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10, 24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26, 19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21, 42,49,35)
7	7	cyclic	4	1	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18, 17,20,21,19)(22,23,25,24,27,28,26)(29,30,32, 31,34,35,33)(36,37,39,38,41,42,40)(43,44,46, 45,48,49,47)
8	8	nilpotent	49	3,4,5	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42, 32,18)(15,27,36,45)(16,44,40,26)(17,39,41, 21)(23,25,47,49)(2,10,5,34)(3,18,6,42)(4,26,7, 44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,1 6,30,40)(14,28,32,46)(15,49,36,25)(19,39,37, 21)(20,24,38,48)(23,27,47,45)
9	14	dihedral	28	2,7	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18, 42)(19,37)(20,38)(21,39)(22,43)(23,47)(24, 48)(25,49)(26,44)(27,45)(28,46),(1,2,4,3,6, 7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21, 19)(22,23,25,24,27,28,26)(29,30,32,31,34,35, 33)(36,37,39,38,41,42,40)(43,44,46,45,48,49, 47)
10	14	dihedral	28	2,6	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18, 42)(19,37)(20,38)(21,39)(22,43)(23,47)(24, 48)(25,49)(26,44)(27,45)(28,46),(1,8,22,15,

					36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
11	49	abelian	1	6,7	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47),(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
12	98	solvable	1	9,10,11	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
13	196	solvable	1	4,12	(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,16,30,40)(14,28,32,46)(15,49,36,25)(19,39,37,21)(20,24,38,48)(23,27,47,45)(1,10,26,18,42,44,34)(2,13,22,17,40,46,35)(3,12,25,21,37,48,29)(4,14,23,20,36,45,33)(5,11,28,16,41,43,31)(6,8,24,19,39,49,30)(7,9,27,15,38,47,32)
14	196	solvable	1	5,12	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,20,30,28,46,12,38)(2,21,32,26,45,8,41)(3,15,34,23,49,11,40)(4,19,31,22,48,9,42)(5,17,29,27,44,14,39)(6,16,35,25,47,10,36)(7,18,33,24,43,13,37)
15	196	solvable	1	3,12	(2,24,5,48)(3,32,6,14)(4,40,7,16)(8,39,29,21)(9,35,33,11)(10,20,34,38)(12,26,30,44)(13,43,31,22)(15,47,36,23)(17,37,41,19)(18,28,42,46)(25,45,49,27)(1,24,40,32,14,16,48)(2,27,36,31,12,18,49)(3,26,39,35,9,20,43)(4,28,37,34,8,17,47)(5,25,42,30,13,15,45)(6,22,38,33,11,21,44)(7,23,41,29,10,19,46)
16	392	*G*	1	8,13,14,15	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(1,23,5,49)(2,34,7,11)(4,15,6,40)(8,32,33,13)(9,21,35,37)(10,41,31,18)(12,26,29,43)(14,45,30,24)(16,36,42,19)(17,47,38,22)(20,25,39,48)(27,44,46,28)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34),(1,31,47,39,21,23,13)(2,34,43,38,19,25,14)(3,33,46,42,16,27,8)(4,35,44,41,15,24,12)(5,32,49,37,20,22,10)(6,29,45,40,18,28,9)(7,30,48,36,17,26,11)
2	98	solvable	abelian	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,17,33,25,49,9,41)(2,20,29,24,47,11,42)(3,19,32,28,44,13,36)(4,21,30,27,43,10,40)(5,18,35,23,48,8,38)(6,15,31,26,46,14,37)(7,16,34,22,45,12,39)
3	196	solvable	cyclic	1,2	(2,34,5,10)(3,42,6,18)(4,44,7,26)(8,17,29,41)(9,22,33,43)(11,13,35,31)(12,40,30,16)(14,46,32,28)(15,25,36,49)(19,21,37,39)(20,48,38,24)(23,45,47,27)(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)(7,35,49,42,21,28,14)
4	196	solvable	cyclic	1,2	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)
5	196	solvable	cyclic	1,2	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	nilpotent	8	*G*	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)
						(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,

7	49	1	abelian	*G*	49	39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,23,39,31,13,21,47)(2,25,38,34,14,19,43)(3,27,42,33,8,16,46)(4,24,41,35,12,15,44)(5,22,37,32,10,20,49)(6,28,40,29,9,18,45)(7,26,36,30,11,17,48)

5.Let *G* be a primitive group of degree 49 with 2 generators. We have $|G| = 392 = 2^3 \times 7^2$.

Generators of *G*:	a_1 = (2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40) (order 8)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

G is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup *D* = [*G*, *G*] is an abelian group of order 49, generated by {(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)} and $G/D \cong C_8$.

Lower central series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	C_8

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	C_8
2	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
3	4	cyclic	49	2	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)
4	7	cyclic	4	1	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
5	7	cyclic	4	1	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,

					34)(7,14,28,21,42,49,35)
6	8	cyclic	49	3	(2,9,38,13,5,33,20,31)(3,17,46,21,6,41,28,39)(4,25,12,23,7,49,30,47)(8,18,37,32,29,42,19,14)(10,35,24,22,34,11,48,43)(15,26,45,40,36,44,27,16)
7	14	dihedral	28	2,4	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
8	14	dihedral	28	2,5	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
9	49	abelian	1	4,5	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
10	98	solvable	1	7,8,9	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
11	196	solvable	1	3,10	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,20,30,28,46,12,38)(2,21,32,26,45,8,41)(3,15,34,23,49,11,40)(4,19,31,22,48,9,42)(5,17,29,27,44,14,39)(6,16,35,25,47,10,36)(7,18,33,24,43,13,37)
12	392	*G*	1	6,11	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,

					31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	196	solvable	cyclic	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,40,14,48,24,32,16)(2,36,12,49,27,31,18)(3,39,9,43,26,35,20)(4,37,8,47,28,34,17)(5,42,13,45,25,30,15)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	cyclic	8	*G*	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)
7	49	1	abelian	*G*	49	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,44,18,10,34,42,26)(2,46,17,13,35,40,22)(3,48,21,12,29,37,25)(4,45,20,14,33,36,23)(5,43,16,11,31,41,28)(6,49,19,8,30,39,24)(7,47,15,9,32,38,27)

6. Let *G* be a primitive group of degree 49 with 4 generators. We have $|G| = 588 = 2^2 \times 3 \times 7^2$.

Generators of *G*:	a_1 = (2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28) (order 4)
	a_2 = (2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46) (order 2)
	a_3 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45) (order 3)
	a_4 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. *G* is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup *D* = [*G*, *G*] is a solvable group of order 147, generated by {(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)} and $G/D \cong C_4$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	C_4

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	C_4
2	49	abelian	solvable	C_3
3	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
3	3	cyclic	49	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
4	4	cyclic	147	2	(2,15,5,36)(3,22,6,43)(4,29,7,8)(9,18,33,42)(10,25,34,49)(11,32,35,14)(12,39,30,21)(13,46,31,28)(16,19,40,37)(17,26,41,44)(20,47,38,23)(24,27,48,45)
5	6	cyclic	49	2,3	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
6	7	cyclic	2	1	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
7	7	cyclic	6	1	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
8	12	solvable	49	4,5	(2,15,5,36)(3,22,6,43)(4,29,7,8)(9,18,33,42)(10,25,34,49)(11,32,35,14)(12,39,30,21)(13,46,31,28)(16,19,40,37)(17,26,41,44)(20,47,38,23)(24,27,48,45)(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
9	14	dihedral	14	2,6	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,

					46)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
10	14	dihedral	42	2,7	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
11	21	solvable	14	3,6	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33), (1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
12	42	solvable	14	5,9,11	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)
13	49	abelian	1	6,7	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
14	98	solvable	1	9,10,13	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
15	147	solvable	1	11,13	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,

					39,43,35)(4,8,28,17,37,47,34)(5,13,25,15, 42,45,30)(6,11,22,21,38,44,33)(7,10,23,19, 41,46,29)
16	196	solvable	3	4,14	(2,15,5,36)(3,22,6,43)(4,29,7,8)(9,18,33,42) (10,25,34,49)(11,32,35,14)(12,39,30,21)(13, 46,31,28)(16,19,40,37)(17,26,41,44)(20,47, 38,23)(24,27,48,45)(1,22,36,29,8,15,43)(2, 23,37,30,9,16,44)(3,24,38,31,10,17,45)(4, 25,39,32,11,18,46)(5,26,40,33,12,19,47)(6, 27,41,34,13,20,48)(7,28,42,35,14,21,49)
17	294	solvable	1	12,14,15	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31) (3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5, 13,25,15,42,45,30)(6,11,22,21,38,44,33)(7, 10,23,19,41,46,29)(2,7,6,5,4,3)(8,15,22,29, 36,43)(9,21,27,33,39,45)(10,16,28,34,40,46) (11,17,23,35,41,47)(12,18,24,30,42,48)(13, 19,25,31,37,49)(14,20,26,32,38,44)
18	588	*G*	1	8,16,17	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28) (11,41,23)(12,42,24)(13,37,25)(14,38,26) (15,43,29)(16,46,34)(17,47,35)(18,48,30) (19,49,31)(20,44,32)(21,45,33)(1,48,11,21) (2,41,9,42)(3,27,12,35)(4,20,8,49)(5,34,10, 28)(6,13,14,7)(15,43,46,18)(16,36,44,39) (17,22,47,32)(19,29,45,25)(23,40,30,38) (24,26,33,31)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17, 16,19,20,18)(22,28,24,23,26,27,25)(29,35,31, 30,33,34,32)(36,42,38,37,40,41,39)(43,49,45, 44,47,48,46)(1,43,15,8,29,36,22)(2,44,16,9, 30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32, 39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41, 27)(7,49,21,14,35,42,28)
2	98	solvable	dihedral	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18, 42)(19,37)(20,38)(21,39)(22,43)(23,47)(24, 48)(25,49)(26,44)(27,45)(28,46)(1,43,15,8, 29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31, 38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40, 26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28) (1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17, 16,19,20,18)(22,28,24,23,26,27,25)(29,35,31, 30,33,34,32)(36,42,38,37,40,41,39)(43,49,45, 44,47,48,46)
3	147	solvable	cyclic	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11, 41,23)(12,42,24)(13,37,25)(14,38,26)(15,43, 29)(16,46,34)(17,47,35)(18,48,30)(19,49,31) (20,44,32)(21,45,33)(1,39,13,47,23,31,21)(2, 38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41, 12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9, 45,28,29,18)(7,36,11,48,26,30,17)
					(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,

4	294	solvable	cyclic	1,2,3	45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)(1,16,32,24,48,14,40)(2,18,31,27,49,12,36)(3,20,35,26,43,9,39)(4,17,34,28,47,8,37)(5,15,30,25,45,13,42)(6,21,33,22,44,11,38)(7,19,29,23,46,10,41)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	4	147	cyclic	4	*G*	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)
3	3	49	cyclic	12	147	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
7	49	1	abelian	*G*	49	(1,36,8,43,22,29,15)(2,37,9,44,23,30,16)(3,38,10,45,24,31,17)(4,39,11,46,25,32,18)(5,40,12,47,26,33,19)(6,41,13,48,27,34,20)(7,42,14,49,28,35,21)(1,48,16,14,32,40,24)(2,49,18,12,31,36,27)(3,43,20,9,35,39,26)(4,47,17,8,34,37,28)(5,45,15,13,30,42,25)(6,44,21,11,33,38,22)(7,46,19,10,29,41,23)

7. Let *G* be a primitive group of degree 49 with 4 generators. We have $|G| = 588 = 2^2 \times 3 \times 7^2$.

Generators of *G*:	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46) (order 2)
	a_3 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45) (order 3)
	a_4 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. *G* is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup *D* = [*G*, *G*] is a solvable group of order 147, generated by {(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)} and $G/D \cong C_2^2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	abelian	C_2^2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	abelian	C_2^2
2	49	abelian	dihedral	C_3
3	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	21	1	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
3	2	cyclic	21	1	(2,15)(3,22)(4,29)(5,36)(6,43)(7,8)(9,21)(10,28)(11,35)(12,42)(13,49)(17,23)(18,30)(19,37)(20,44)(25,31)(26,38)(27,45)(33,39)(34,46)(41,47)
4	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
5	3	cyclic	49	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
6	4	abelian	147	2,3,4	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46), (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
7	6	dihedral	49	2,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
8	6	dihedral	49	3,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,15)(3,22)(4,29)(5,36)(6,43)(7,8)(9,21)(10,28)(11,35)(12,42)(13,49)(17,23)(18,30)(19,37)(20,44)(25,31)(26,38)(27,45)(33,39)

					(34,46)(41,47)
9	6	cyclic	49	4,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
10	7	cyclic	2	1	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
11	7	cyclic	3	1	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
12	7	cyclic	3	1	(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)
13	12	dihedral	49	6,7,8,9	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)
14	14	dihedral	3	3,12	(2,15)(3,22)(4,29)(5,36)(6,43)(7,8)(9,21)(10,28)(11,35)(12,42)(13,49)(17,23)(18,30)(19,37)(20,44)(25,31)(26,38)(27,45)(33,39)(34,46)(41,47)(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)
15	14	dihedral	3	2,11	(2,36)(3,43)(4,8)(5,15)(6,22)(7,29)(9,39)(10,46)(12,18)(13,25)(14,32)(16,40)(17,47)(20,26)(21,33)(23,41)(24,48)(28,34)(30,42)(31,49)(38,44)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
					(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)

16	14	dihedral	14	4,10	(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
17	14	cyclic	21	2,12	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,49,17,9,33,41,25)(2,47,20,11,29,42,24)(3,44,19,13,32,36,28)(4,43,21,10,30,40,27)(5,48,18,8,35,38,23)(6,46,15,14,31,37,26)(7,45,16,12,34,39,22)
18	14	cyclic	21	3,11	(2,15)(3,22)(4,29)(5,36)(6,43)(7,8)(9,21)(10,28)(11,35)(12,42)(13,49)(17,23)(18,30)(19,37)(20,44)(25,31)(26,38)(27,45)(33,39)(34,46)(41,47)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
19	14	dihedral	21	4,12	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)
20	14	dihedral	21	4,11	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
21	21	solvable	14	5,10	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,

					49,45,44,47,48,46)
22	28	dihedral	21	6,14,17,19	(1,9)(3,44)(4,30)(5,23)(6,37)(7, 16)(10,43)(11,29)(12,22)(13,36)(14,15)(17,49)(18,35)(19,28)(20, 42)(24,47)(25,33)(27,40)(31,46)(34,39)(38,48)(1,41,9,49,25,33,17)(2,48,11,35,24,5,20,8,42,23,47,18, 29,38)(3,6,13,14,28,26,19,15,36,37, 44,46,32,31)(4,34,10,7,27,12,21,22, 40,16,43,39,30,45)
23	28	dihedral	21	6,15,18,20	(1,16)(3,9)(4,37)(5,30)(6,44)(7,23)(8,17)(11,38)(12,31)(13,45)(14,24)(18,36)(19,29)(20,43)(21,22)(25, 42)(26,35)(27,49)(32,40)(34,47)(41,46)(1,48,16,14,32,40,24)(2,13, 18,42,31,5,27,15,49,30,12,25,36, 45)(3,6,20,21,35,33,26,22,43,44,9, 11,39,38)(4,41,17,7,34,19,28,29,47, 23,8,46,37,10)
24	42	solvable	14	9,16,21	(1,7,3,2,5,6,4)(8,14,10,9,12,13, 11)(15,2117,16,19,20,18)(22,28, 24,23,26,27,25)(29,35,31,30,33, 34,32)(36,42,38,37,40,41,39)(43, 49,45,44,47,48,46)(2,7,6,5,4,3)(8, 15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41, 47)(12,18,24,30,42,48)(13,19,25,31, 37,49)(14,20,26,32,38,44)
25	49	abelian	1	10,11,12	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,2117,16,19,20,18)(22,28,24,23, 26,27,25)(29,35,31,30,33,34,32)(36, 42,38,37,40,41,39)(43,49,45,44,47, 48,46)(1,14,24,16,40,48,32)(2,12,27, 18,36,49,31)(3,9,26,20,39,43,35)(4,8, 28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41, 46,29)
26	98	solvable	1	16,19,20,25	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15, 36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24, 48)(25,49)(26,44)(27,45)(28,46)(1,14,24,16,40,48,32)(2,12,27,18, 36,49,31)(3,9,26,20,39,43,35)(4,8, 28,17,37,47,34)(5,13,25,15,42,45, 30)(6,11,22,21,38,44,33)(7,10,23, 19,41,46,29)(1,7,3,2,5,6,4)(8,14,10, 9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31, 30,33,34,32)(36,42,38,37,40,41, 39)(43,49,45,44,47,48,46)
					(1,28,38,30,12,20,46)(2,26,41,32, 8,21,45)(3,23,40,34,11,15,49)(4, 22,42,31,9,19,48)(5,27,39,29,14,

27	98	solvable	3	15,17,25	17,44)(6,25,36,35,10,16,47)(7,24,37,33,13,18,43)(1,33,49,41,17,25,9)(2,5,47,48,20,18,11,8,29,35,42,38,24,23)(3,26,44,6,19,46,13,15,32,14,36,31,28,37)(4,12,43,34,21,39,10,22,30,7,40,45,27,16)
28	98	solvable	3	14,18,25	(1,35,45,37,19,27,11)(2,33,48,39,15,28,10)(3,30,47,41,18,22,14)(4,29,49,38,16,26,13)(5,34,46,36,21,24,9)(6,32,43,42,17,23,12)(7,31,44,40,20,25,8)(1,40,14,48,24,32,16)(2,5,12,13,27,25,18,15,36,42,49,45,31,30)(3,33,9,6,26,11,20,22,39,21,43,38,35,44)(4,19,8,41,28,46,17,29,37,7,47,10,34,23)
29	147	solvable	1	21,25	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
30	196	solvable	3	22,23,26,27,28	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,43,49,21,17,10,9,30,33,40,41,27,25,4)(2,29,47,42,20,24,11)(3,8,44,35,19,38,13,23,32,5,36,48,28,18)(6,22,46,7,15,45,14,16,31,12,37,34,26,39)
31	294	solvable	1	24,26,29	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)
32	294	solvable	1	7,27,29	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,43,49,21,17,10,9,30,33,40,41,27,25,4)(2,29,47,42,20,24,11)(3,8,44,35,19,38,13,23,32,5,36,48,28,18)(6,22,46,7,15,45,14,16,31,12,37,34,26,39)
					(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,

33	294	solvable	1	8,28,29	40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,8,14,28,24,17,16,37,40,47,48,34,32,4)(2,36,12,49,27,31,18)(3,15,9,42,26,45,20,30,39,5,43,13,35,25)(6,29,11,7,22,10,21,23,38,19,44,41,33,46)
34	588	*G*	1	13,30,31,32,33	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,7,4,2,3,5)(8,21,25,30,38,47)(9,17,26,29,42,46)(10,19,22,35,39,44)(11,16,24,33,36,49)(12,15,28,32,37,45)(13,20,27,34,41,48)(14,18,23,31,40,43)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	dihedral	--	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
2	98	solvable	dihedral	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
3	147	solvable	abelian	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
4	294	solvable	cyclic	1,3	(1,26,20,45,30,8,28,41,46,2,12,21,38,32)(3,33,15,24,34,43,23,13,49,37,11,7,40,18)(4,5,19,17,31,29,22,27,48,44,9,14,42,39)(6,47,16,10,35,36,25)(1,47,11)(2,45,13)(3,48,9)(4,43,12)(5,46,8)(6,44,10)(7,49,14)(15,40,32)(16,38,34)(17,41,30)(18,36,33)(19,39,29)(20,37,31)(21,42,35)(22,26,25)(23,24,27)

5	294	solvable	cyclic	1,2,3	(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33, 39,45)(10,16,28,34,40,46)(11,17,23,35,41, 47)(12,18,24,30,42,48)(13,19,25,31,37,49) (14,20,26,32,38,44)(1,39,13,47,23,31,21) (2,38,14,43,25,34,19)(3,42,8,46,27,33,16) (4,41,12,44,24,35,15)(5,37,10,49,22,32,20) (6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
6	294	solvable	cyclic	1,3	(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40) (11,23,41)(12,24,42)(13,25,37)(14,26,38) (15,29,43)(16,34,46)(17,35,47)(18,30,48) (19,31,49)(20,32,44)(21,33,45)(1,36,41,13, 9,44,49,28,25,32,33,19,17,3)(2,43,42,27,11, 30,47,21,24,4,29,40,20,10)(5,15,38,6,8,37, 48,14,23,46,35,26,18,31)(7,22,39,34,12,16, 45)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	4	147	abelian	4	*G*	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34) (11,35)(12,30)(13,31)(14,32)(15, 36)(16,40)(17,41)(18,42)(19,37) (20,38)(21,39)(22,43)(23,47)(24, 48)(25,49)(26,44)(27,45)(28,46) (2,8)(3,15)(4,22)(5,29)(6,36)(7, 43)(10,16)(11,23)(12,30)(13,37) (14,44)(18,24)(19,31)(20,38)(21, 45)(26,32)(27,39)(28,46)(34,40) (35,47)(42,48)
3	3	49	cyclic	12	147	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10, 40,28)(11,41,23)(12,42,24)(13,37, 25)(14,38,26)(15,43,29)(16,46,34) (17,47,35)(18,48,30)(19,49,31)(20, 44,32)(21,45,33)
7	49	1	abelian	*G*	49	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11) (15,21,17,16,19,20,18)(22,28,24,23, 26,27,25)(29,35,31,30,33,34,32)(36, 42,38,37,40,41,39)(43,49,45,44,47, 48,46)(1,43,15,8,29,36,22)(2,44,16, 9,30,37,23)(3,45,17,10,31,38,24)(4, 46,18,11,32,39,25)(5,47,19,12,33,40, 26)(6,48,20,13,34,41,27)(7,49,21,14, 35,42,28)

8. Let G be a primitive group of degree 49 with 2 generators. We have $|G| = 588 = 2^2 \times 3 \times 7^2$.

Generators of *G*:	a_1 = (2,30,3,38,4,46,5,12,6,20,7,28)(8,35,15,37, 22,45,29,11,36,19,43,27)(9,47,17,13,25,21,33, 23,41,31,49,39)(10,16,18,24,26,32,34,40,42,48, 44,14) (order 12)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5, 12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35) (order 7)

G is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup $D = [G, G]$ is an abelian group of order 49, generated by {(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)(1,49,17,9,33,41,25)(2,47,20,11,29,42,24)(3,44,19,13,32,36,28)(4,43,21,10,30,40,27)(5,48,18,8,35,38,23)(6,46,15,14,31,37,26)(7,45,16,12,34,39,22)} and $G/D \cong C_3 \times C_4$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	$C_3 \times C_4$

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	$C_3 \times C_4$
2	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
3	3	cyclic	49	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
4	4	cyclic	49	2	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)
5	6	cyclic	49	2,3	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
6	7	cyclic	2	1	(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)
7	7	cyclic	2	1	(1,32,48,40,16,24,14)(2,31,49,36,18,27,12)(3,35,43,39,20,26,9)(4,34,47,37,17,28,8)(5,30,45,42,15,25,13)(6,

					33,44,38,21,22,11)(7,29,46,41,19, 23,10)
8	7	cyclic	2	1	(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26, 23,24,28)(29,32,34,33,30,31,35)(36, 39,41,40,37,38,42)(43,46,48,47,44, 45,49)
9	7	cyclic	2	1	(1,11,27,19,37,45,35)(2,10,28,15,39, 48,33)(3,14,22,18,41,47,30)(4,13,26, 16,38,49,29)(5,9,24,21,36,46,34)(6, 12,23,17,42,43,32)(7,8,25,20,40,44, 31)
10	12	cyclic	49	4,5	(2,30,3,38,4,46,5,12,6,20,7,28)(8,35, 15,37,22,45,29,11,36,19,43,27)(9,47, 17,13,25,21,33,23,41,31,49,39)(10,16, 18,24,26,32,34,40,42,48,44,14)
11	14	dihedral	14	2,9	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11, 35)(12,30)(13,31)(14,32)(15,36)(16, 40)(17,41)(18,42)(19,37)(20,38)(21, 39)(22,43)(23,47)(24,48)(25,49)(26, 44)(27,45)(28,46)(1,11,27,19,37,45, 35)(2,10,28,15,39,48,33)(3,14,22,18, 41,47,30)(4,13,26,16,38,49,29)(5,9, 24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31)
12	14	dihedral	14	2,6	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15, 36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,21, 31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44, 12,41)(5,20,32,22,49,10,37)(6,18,29, 28,45,9,40)(7,17,30,26,48,11,36)
13	14	dihedral	14	2,7	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11, 35)(12,30)(13,31)(14,32)(15,36)(16, 40)(17,41)(18,42)(19,37)(20,38)(21, 39)(22,43)(23,47)(24,48)(25,49)(26, 44)(27,45)(28,46)(1,32,48,40,16,24, 14)(2,31,49,36,18,27,12)(3,35,43,39, 20,26,9)(4,34,47,37,17,28,8)(5,30,45, 42,15,25,13)(6,33,44,38,21,22,11)(7, 29,46,41,19,23,10)
14	14	dihedral	14	2,8	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11, 35)(12,30)(13,31)(14,32)(15,36)(16, 40)(17,41)(18,42)(19,37)(20,38)(21, 39)(22,43)(23,47)(24,48)(25,49)(26, 44)(27,45)(28,46)(1,4,6,5,2,3,7)(8, 11,13,12,9,10,14)(15,18,20,19,16, 17,21)(22,25,27,26,23,24,28)(29, 32,34,33,30,31,35)(36,39,41,40,37, 38,42)(43,46,48,47,44,45,49)
					(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,

15	21	solvable	14	3,9	23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31)
16	21	solvable	14	3,7	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,32,48,40,16,24,14)(2,31,49,36,18,27,12)(3,35,43,39,20,26,9)(4,34,47,37,17,28,8)(5,30,45,42,15,25,13)(6,33,44,38,21,22,11)(7,29,46,41,19,23,10)
17	21	solvable	14	3,8	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
18	21	solvable	14	3,6	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)
19	42	solvable	14	5,14,17	(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
20	42	solvable	14	5,12,18	(1,21,31,23,47,13,39)(2,19,34,25,43,14,8)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
					(1,32,48,40,16,24,14)(2,31,49,36,18,

21	42	solvable	14	5,13,16	27,12)(3,35,43,39,20,26,9)(4,34,47,37,17,28,8)(5,30,45,42,15,25,13)(6,33,44,38,21,22,11)(7,29,46,41,19,23,10)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,3931,23,21)(14,48,40,32,24,16)
22	42	solvable	14	5,11,15	(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
23	49	abelian	1	6,7,8,9	(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31)
24	98	solvable	1	11,12,13,14,23	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46),(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
25	147	solvable	1	15,16,17,18,23	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
					(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,

26	196	solvable	1	4,24	22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)
27	294	solvable	1	19,20,21,22,24,25	(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
28	588	*G*	1	10,26,27	(2,30,3,38,4,46,5,12,6,20,7,28)(8,35,15,37,22,45,29,11,36,19,43,27)(9,47,17,13,25,21,33,23,41,31,49,39)(10,16,18,24,26,32,34,40,42,48,44,14)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	294	solvable	cyclic	(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)(1,26,42,34,10,18,44)(2,22,40,35,13,17,46)(3,25,37,29,12,21,48)(4,23,36,33,14,20,45)(5,28,41,31,11,16,43)(6,24,39,30,8,19,49)(7,27,38,32,9,15,47)
2	196	solvable	cyclic	(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)(1,23,5,49)(2,34,7,11)(4,15,6,40)(8,32,33,13)(9,21,35,37)(10,41,31,18)(12,26,29,43)(14,45,30,24)(16,36,42,19)(17,47,38,22)(20,25,39,48)(27,44,46,28)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	4	49	cyclic	12	196	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)
3	3	49	] cyclic	12	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)

						(19,35,45)(20,30,46)(21,31,47)
7	49	1	abelian	*G*	49	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31) (3,9,26,20,39,43,35)(4,8,28,17,37,47,34) (5,13,25,15,42,45,30)(6,11,22,21,38,44,33) (7,10,23,19,41,46,29)(1,15,29,22,43,8,36) (2,16,30,23,44,9,37)(3,17,31,24,45,10,38) (4,18,32,25,46,11,39)(5,19,33,26,47,12,40) (6,20,34,27,48,13,41)(7,21,35,28,49,14,42)

9. Let *G* be a primitive group of degree 49 with 3 generators. We have $|G| = 784 = 2^4 \times 7^2$.

Generators of *G*:	a_1 = (2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38) (14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47) (32,42)(40,44) (order 2)
	a_2 = (2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23) (8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22) (15,44,45,16,36,26,27,40) (order 8)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31) (4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller set with 2 elements. *G* is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 196, generated by {(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32, 18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,21,31,23,47,13,39)(2,19,34,25,43,14, 38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30, 26,48,11,36)} and $G/D \cong C_2^2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	C_2^2
2	98	solvable	nilpotent	C_2
3	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	C_2^2
2	49	abelian	nilpotent	C_4
3	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	28	1	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14, 29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38) (21,41)(22,48)(23,49)(24,43)(25,47)(26,45) (27,44)(28,46)
3	2	cyclic	28	1	(2,9)(3,17)(4,25)(5,33)(6,41)(7,49)(8,29)(10, 24)(12,47)(13,20)(14,42)(15,36)(16,44)(18,32) (21,28)(22,43)(23,30)(26,40)(31,38)(34,48) (39,46)
					(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,

4	2	cyclic	49	1	30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
5	4	cyclic	49	4	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)
6	4	abelian	98	3,4	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(2,9)(3,17)(4,25)(5,33)(6,41)(7,49)(8,29)(10,24)(12,47)(13,20)(14,42)(15,36)(16,44)(18,32)(21,28)(22,43)(23,30)(26,40)(31,38)(34,48)(39,46)
7	4	abelian	98	2,4	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(2,5)(3,6)(4,7)(8,14)(9,13)(10,11)(15,16)(17,21)(18,19)(22,24)(23,25)(26,27)(29,32)(31,33)(34,35)(36,40)(37,42)(39,41)(43,48)(44,45)(47,49)
8	7	cyclic	4	1	(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)
9	7	cyclic	4	1	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
10	8	cyclic	49	5	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)
11	8	nilpotent	49	5,6	(2,13)(3,21)(4,23)(5,31)(6,39)(7,47)(8,19)(9,38)(10,34)(11,43)(12,25)(15,27)(17,46)(18,42)(20,33)(22,35)(26,44)(28,41)(29,37)(30,49)(36,45),(2,9)(3,17)(4,25)(5,33)(6,41)(7,49)(8,29)(10,24)(12,47)(13,20)(14,42)(15,36)(16,44)(18,32)(21,28)(22,43)(23,30)(26,40)(31,38)(34,48)(39,46)
12	8	nilpotent	49	5,7	(2,20)(3,28)(4,30)(5,38)(6,46)(7,12)(8,18)(9,33)(10,43)(11,24)(14,37)(15,26)(16,45)(17,41)(19,32)(22,34)(25,49)(27,40)(29,42)(35,48)(36,44)(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)
13	14	dihedral	4	2,8	(2,5)(3,6)(4,7)(8,14)(9,13)(10,11)(15,16)(17,21)(18,19)(22,24)(23,25)(26,27)(29,32)(31,33)(34,35)(36,40)(37,42)(39,41)(43,48)(44,45)(47,49)(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)

14	14	dihedral	4	3,9	(2,9)(3,17)(4,25)(5,33)(6,41)(7,49)(8,29)(10,24)(12,47)(13,20)(14,42)(15,36)(16,44)(18,32)(21,28)(22,43)(23,30)(26,40)(31,38)(34,48)(39,46)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
15	14	cyclic	28	3,9	(2,9)(3,17)(4,25)(5,33)(6,41)(7,49)(8,29)(10,24)(12,47)(13,20)(14,42)(15,36)(16,44)(18,32)(21,28)(22,43)(23,30)(26,40)(31,38)(34,48)(39,46)(1,45,19,11,35,37,27)(2,48,15,10,33,39,28)(3,47,18,14,30,41,22)(4,49,16,13,29,38,26)(5,46,21,9,34,36,24)(6,43,17,12,32,42,23)(7,44,20,8,31,40,25)
16	14	dihedral	28	4,8	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)
17	14	dihedral	28	4,9	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
18	14	cyclic	28	2,8	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46),(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)
19	16	nilpotent	49	10,11,12	(2,9)(3,17)(4,25)(5,33)(6,41)(7,49)(8,29)(10,24)(12,47)(13,20)(14,42)(15,36)(16,44)(18,32)(21,28)(22,43)(23,30)(26,40)(31,38)(34,48)(39,46)(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)
20	28	dihedral	28	7,13,16,18	(1,4)(3,5)(6,7)(9,12)(10,13)(11,14)(15,17)(16,18)(19,20)(22,28)(23,27)(24,25)(29,34)(30,31)(33,35)(36,37)(38,42)(39,40)(43,47)(44,49)(46,48)(1,4,6,5,2,3,7)(8,34,13,30,9,35,14,32,11,33,12,31,10,29)(15,37,20,42,16,39,21,40,18,38,19,36,17,41)(22,47,27,45,23,43,28,48,25,44,26,49,24,46)
21	28	dihedral	28	6,14,15,17	(1,11)(3,47)(4,29)(5,24)(6,42)(7,20)(8,25)(10,33)(12,17)(13,49)(14,41)(15,39)(18,22)(19,45)(21,34)(26,38)(27,35)(28,48)(31,40)(32,43)(36,46)(1,37,11,45,27,35,19)(2,46,10,

					34,28,5,15,9,39,24,48,21,33,36)(3,6,14,12,22, 23,18,17,41,42,47,43,30,32)(4,31,13,7,26,8, 16,25,38,20,49,40,29,44)
22	49	abelian	1	8,9	(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21, 20,17,18,16)(22,26,28,27,24,25,23)(29,33,35, 34,31,32,30)(36,40,42,41,38,39,37)(43,47,49, 48,45,46,44)(1,9,25,17,41,49,33)(2,11,24,20, 42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40, 43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46, 31)(7,12,22,16,39,45,34)
23	98	solvable	1	16,17,22	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18, 42)(19,37)(20,38)(21,39)(22,43)(23,47)(24, 48)(25,49)(26,44)(27,45)(28,46)(1,9,25,17, 41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36, 44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48, 35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34) (1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21, 20,17,18,16)(22,26,28,27,24,25,23)(29,33,35, 34,31,32,30)(36,40,42,41,38,39,37)(43,47,49, 48,45,46,44)
24	98	solvable	4	13,18,22	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16, 21,18,19,17)(22,27,23,28,25,26,24)(29,34,30, 35,32,33,31)(36,41,37,42,39,40,38)(43,48,44, 49,46,47,45)(1,12,28,20,38,46,30)(2,14,26,17, 41,44,32,5,8,27,21,39,45,29)(3,10,23,16,40,47, 34,6,11,25,15,36,49,35)(4,13,22,18,42,43,31,7, 9,24,19,37,48,33)
25	98	solvable	4	14,15,22	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10, 24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26, 19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21, 42,49,35)(1,19,35,27,45,11,37)(2,36,33,21,48, 24,39,9,15,5,28,34,10,46)(3,32,30,43,47,42,41, 17,18,23,22,12,14,6)(4,44,29,40,49,20,38,25,1 6,8,26,7,13,31)
26	196	solvable	1	5,23	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37) (9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42, 32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21) (23,25,47,49)(1,38,12,46,28,30,20)(2,41,8,45, 26,32,21)(3,40,11,49,23,34,15)(4,42,9,48,22, 31,19)(5,39,14,44,27,29,17)(6,36,10,47,25,35, 16)(7,37,13,43,24,33,18)
27	196	solvable	2	20,23,24	(1,4,6,5,2,3,7)(8,34,13,30,9,35,14,32,11,33, 12,31,10,29)(15,37,20,42,16,39,21,40,18,38, 19,36,17,41)(22,47,27,45,23,43,28,48,25,44, 26,49,24,46)(1,46,20,12,30,38,28)(2,44,21, 14,32,39,26,5,45,17,8,29,41,27)(3,47,15,10, 34,36,23,6,49,16,11,35,40,25)(4,43,19,13,31, 37,22,7,48,18,9,33,42,24)
28	196	solvable	2	21,23,25	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18, 42)(19,37)(20,38)(21,39)(22,43)(23,47)(24, 48)(25,49)(26,44)(27,45)(28,46)(1,33,35,48, 45,39,37,15,19,28,27,10,11,2)(3,25,30,8,47,7,

					41,31,18,44,22,40,14,20)(4,9,29,5,49,34,38, 46,16,36,26,21,13,24)(6,17,32,23,43,12,42)
29	392	solvable	1	12,26,27	(2,20)(3,28)(4,30)(5,38)(6,46)(7,12)(8,18)(9, 33)(10,43)(11,24)(14,37)(15,26)(16,45)(17, 41)(19,32)(22,34)(25,49)(27,40)(29,42)(35, 48)(36,44)(1,17,30,27,46,14,38,5,20,29,28, 44,12,39)(2,18,32,24,45,13,41,7,21,33,26,43, 8,37)(3,15,34,23,49,11,40)(4,16,31,25,48,10, 42,6,19,35,22,47,9,36)
30	392	solvable	1	11,26,28	(2,9)(3,17)(4,25)(5,33)(6,41)(7,49)(8,29)(10, 24)(12,47)(13,20)(14,42)(15,36)(16,44)(18,32) (21,28)(22,43)(23,30)(26,40)(31,38)(34,48) (39,46)(1,31,32,49,48,36,40,18,16,27,24,12,14, 2)(3,23,35,10,43,7,39,29,20,46,26,41,9,19)(4, 13,34,5,47,30,37,45,17,42,28,15,8,25)(6,21,33, 22,44,11,38)
31	392	solvable	1	10,26	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21) (4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19, 32)(10,11,24,43,34,35,48,22)(15,44,45,16,36, 26,27,40)(1,9,25,17,41,49,33)(2,11,24,20,42, 47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43, 30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31) (7,12,22,16,39,45,34)
32	784	*G*	1	19,29,30,31	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14, 29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38) (21,41)(22,48)(23,49)(24,43)(25,47)(26,45) (27,44)(28,46),(1,38,43,24,15,3,8,45,29,17,36, 10,22,31)(2,18,44,11,16,32,9,39,30,25,37,4,23, 46)(5,48,47,20,19,13,12,34,33,41,40,27,26,6) (7,35,49,42,21,28,14)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19, 18,21,16,20)(22,24,26,25,28,23,27)(29,31, 33,32,35,30,34)(36,38,40,39,42,37,41)(43, 45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16, 30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32, 25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27, 48,13,41)(7,21,35,28,49,14,42)
2	98	solvable	nilpotent	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35) (12,30)(13,31)(14,32)(15,36)(16,40)(17,41) (18,42)(19,37)(20,38)(21,39)(22,43)(23,47) (24,48)(25,49)(26,44)(27,45)(28,46)(1,15,29, 22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24, 45,10,38)(4,18,32,25,46,11,39)(5,19,33,26, 47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49, 14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15, 17,19,18,21,16,20)(22,24,26,25,28,23,27)(29, 31,33,32,35,30,34)(36,38,40,39,42,37,41)(43, 45,47,46,49,44,48)
					(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19) (9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18, 32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)

3	196	solvable	abelian	1,2	(23,49,47,25)(1,46,20,12,30,38,28)(2,45,21,8,32,41,26)(3,49,15,11,34,40,23)(4,48,19,9,31,42,22)(5,44,17,14,29,39,27)(6,47,16,10,35,36,25)(7,43,18,13,33,37,24)
4	392	solvable	cyclic	1,2,3	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
5	392	solvable	cyclic	1,2,3	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,46)(2,45)(3,49)(4,48)(5,44)(6,47)(7,43)(8,41)(9,42)(10,36)(11,40)(12,38)(13,37)(14,39)(15,23)(16,25)(17,27)(18,24)(19,22)(20,28)(21,26)
6	392	solvable	cyclic	1,2,3	(2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44)(1,11,18,41,34,5,26,16,44,31,10,28,42,43)(2,7,17,15,35,32,22,27,46,47,13,9,40,38)(3,48,21,12,29,37,25)(4,30,20,24,33,49,23,8,45,39,14,6,36,19)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	16	49	nilpotent	16	*G*	(2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44)(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)
7	49	1	abelian	*G*	49	(1,22,36,29,8,15,43)(2,23,37,30,9,16,44)(3,24,38,31,10,17,45)(4,25,39,32,11,18,46)(5,26,40,33,12,19,47)(6,27,41,34,13,20,48)(7,28,42,35,14,21,49)(1,38,12,46,28,30,20)(2,41,8,45,26,32,21)(3,40,11,49,23,34,15)(4,42,9,48,22,31,19)(5,39,14,44,27,29,17)(6,36,10,47,25,35,16)(7,37,13,43,24,33,18)

10. Let *G* be a primitive group of degree 49 with 2 generators. We have $|G| = 784 = 2^4 \times 7^2$.

Generators of *G*:	a_1 = (2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44) (order 16)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

G is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial. The derived subgroup *D* = [*G*, *G*] is an abelian group of order 49, generated by

{(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)(1,28,38,30,12,20,46)(2,26,41,32,8,21,45)(3,23,40,34,11,15,49)(4,22,42,31,9,19,48)(5,27,39,29,14,17,44)(6,25,36,35,10,16,47)(7,24,37,33,13,18,43)} and $G/D \cong C_{16}$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	C_{16}

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	C_{16}
2	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
3	4	cyclic	49	2	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)
4	7	cyclic	8	1	(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
5	8	cyclic	49	3	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)
6	14	dihedral	56	2,4	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
7	16	cyclic	49	5	(2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44)
8	49	abelian	1	4	(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)

9	98	solvable	1	6,8	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
10	196	solvable	1	3,9	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)
11	392	solvable	1	5,10	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)
12	784	*G*	1	7,11	(2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	392	solvable	cyclic	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	16	49	cyclic	16	*G*	(2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44)
7	49	1	abelian	*G*	49	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,23,39,31,13,21,47)

						(2,25,38,34,14,19,43)(3,27,42,33,8,16,46) (4,24,41,35,12,15,44)(5,22,37,32,10,20,49) (6,28,40,29,9,18,45)(7,26,36,30,11,17,48)

11. Let G be a primitive group of degree 49 with 3 generators. We have $|G| = 784 = 2^4 \times 7^2$.

Generators of G:	a_1 = (2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35) (10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47) (17,19,41,37)(18,46,42,28)(25,27,49,45) (order 4)
	a_2 = (2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21) (4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32) (10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40) (order 8)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46,32) (5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 196, generated by {(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,28,38,30,12,20,46)(2,26,41,32,8,21,45)(3,23,40,34,11,15,49)(4,22,42,31,9,19,48)(5,27,39,29,14,17,44)(6,25,36,35,10,16,47)(7,24,37,33,13,18,43)} and $G/D \cong C_2^2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	C_2^2
2	98	solvable	nilpotent	C_2
3	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	C_2^2
2	49	abelian	nilpotent	C_4
3	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
3	4	cyclic	49	2	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)
4	4	cyclic	98	2	(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,16,30,40)(14,28,32,46)(15,49,36,25)(19,39,37,21)(20,24,38,48)(23,27,47,45)

5	4	cyclic	98	2	(2,22,5,43)(3,29,6,8)(4,36,7,15)(9,24,33,48)(10,31,34,13)(11,38,35,20)(12,45,30,27)(14,17,32,41)(16,25,40,49)(18,39,42,21)(19,46,37,28)(23,26,47,44)
6	7	cyclic	8	1	(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
7	8	cyclic	49	3	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,6,36,26,27,40)
8	8	nilpotent	49	3,4	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,16,30,40)(14,28,32,46)(15,49,36,25)(19,39,37,21)(20,24,38,48)(23,27,47,45)
9	8	nilpotent	49	3,5	(2,35,5,11)(3,37,6,19)(4,45,7,27)(8,28,29,46)(9,10,33,34)(12,15,30,36)(13,48,31,24)(14,39,32,21)(16,47,40,23)(17,18,41,42)(20,22,38,43)(25,26,49,44)(2,22,5,43)(3,29,6,8)(4,36,7,15)(9,24,33,48)(10,31,34,13)(11,38,35,20)(12,45,30,27)(14,17,32,41)(16,25,40,49)(18,39,42,21)(19,46,37,28)(23,26,47,44)
10	14	dihedral	56	2,6	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
11	16	nilpotent	49	7,8,9	(2,22,5,43)(3,29,6,8)(4,36,7,15)(9,24,33,48)(10,31,34,13)(11,38,35,20)(12,45,30,27)(14,17,32,41)(16,25,40,49)(18,39,42,21)(19,46,37,28)(23,26,47,44)(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,16,30,40)(14,28,32,46)(15,49,36,25)(19,39,37,21)(20,24,38,48)(23,27,47,45)
12	49	abelian	1	6	(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)
13	98	solvable	1	10,12	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,

					29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
14	196	solvable	1	3,13	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)
15	196	solvable	2	4,13	(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,16,30,40)(14,28,32,46)(15,49,36,25)(19,39,37,21)(20,24,38,48)(23,27,47,45)(1,26,42,34,10,18,44)(2,22,40,35,13,17,46)(3,25,37,29,12,21,48)(4,23,36,33,14,20,45)(5,28,41,31,11,16,43)(6,24,39,30,8,19,49)(7,27,38,32,9,15,47)
16	196	solvable	2	5,13	(2,22,5,43)(3,29,6,8)(4,36,7,15)(9,24,33,48)(10,31,34,13)(11,38,35,20)(12,45,30,27)(14,17,32,41)(16,25,40,49)(18,39,42,21)(19,46,37,28)(23,26,47,44)(1,36,8,43,22,29,15)(2,37,9,44,23,30,16)(3,38,10,45,24,31,17)(4,39,11,46,25,32,18)(5,40,12,47,26,33,19)(6,41,13,48,27,34,20)(7,42,14,49,28,35,21)
17	392	solvable	1	8,14,15	(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,16,30,40)(14,28,32,46)(15,49,36,25)(19,39,37,21)(20,24,38,48)(23,27,47,45),(1,12,13,7)(2,46,10,18)(3,30,9,24)(4,28,11,33)(5,38,14,37)(6,20,8,43)(15,32,48,25)(16,21,45,47)(17,41,44,36)(19,26,49,35)(22,23,34,31)(27,40,29,42)
18	392	solvable	1	9,14,16	(2,22,5,43)(3,29,6,8)(4,36,7,15)(9,24,33,48)(10,31,34,13)(11,38,35,20)(12,45,30,27)(14,17,32,41)(16,25,40,49)(18,39,42,21)(19,46,37,28)(23,26,47,44)(1,36,47,12)(2,21,49,23)(3,9,45,42)(4,24,48,17)(5,46,43,6)(7,34,44,32)(8,35,40,30)(10,19,38,22)(11,37,41,14)(13,27,39,18)(15,16,26,28)(20,29,25,33)
19	392	solvable	1	7,14	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28 21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)
20	784	*G*	1	11,17,18,19	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(1,44,20,35,21,48,2,36)(3,28,12,15,18,8,26,7)(4,19,38,41,17,5,32,30)(6,13,46,24,16,23,45,11)(9,14,37,33,27,22,34,40)(10,31,43,42,25,39,49,29)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	32	solvable	cyclic	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,28,48,19)(2,42,45,12)(3,14,44,40)(4,35,46,33)(5,7,49,47)(6,21,43,26)(8,23,41,17)(9,37,38,10)(11,30,39,31)(13,16,36,24)(15,22,27,20)(18,29,25,34)
2	392	solvable	cyclic	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)(1,28,38,30,12,20,46)(2,26,41,32,8,21,45)(3,23,40,34,11,15,49)(4,22,42,31,9,19,48)(5,27,39,29,14,17,44)(6,25,36,35,10,16,47)(7,24,37,33,13,18,43)
3	392	solvable	cyclic	(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,16,30,40)(14,28,32,46)(15,49,36,25)(19,39,37,21)(20,24,38,48)(23,27,47,45)(1,9,24,11)(2,35,25,21)(3,26,22,6)(4,39,23,44)(5,20,27,33)(7,43,28,38)(8,40,10,48)(12,32,13,16)(15,18,31,30)(17,36,29,45)(19,49,34,42)(37,41,46,47)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	16	49	nilpotent	16	*G*	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)
7	49	1	abelian	*G*	49	(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)(1,44,18,10,34,42,26)(2,46,17,13,35,40,22)(3,48,21,12,29,37,25)(4,45,20,14,33,36,23)(5,43,16,11,31,41,28)(6,49,19,8,30,39,24)(7,47,15,9,32,38,27)

12. Let *G* be a primitive group of degree 49 with 4 generators. We have $|G| = 882 = 2\times3^2\times7^2$.

Generators of *G*:	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45) (order 3)
	a_3 = (2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (order 3)

	a_4 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46,32) (5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. *G* is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 147, generated by {(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)} and $G/D \cong C_2 \times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	$C_2 \times C_3$

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	$C_2 \times C_3$
2	49	abelian	solvable	C_3
3	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	21	1	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
3	3	cyclic	14	1	(8,36,22)(9,37,23)(10,38,24)(11,39,25)(12,40,26)(13,41,27)(14,42,28)(15,43,29)(16,44,30)(17,45,31)(18,46,32)(19,47,33)(20,48,34)(21,49,35)
4	3	cyclic	49	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
5	3	cyclic	49	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
6	6	dihedral	49	2,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
					(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)

7	6	cyclic	147	2,4	(20,30,46)(21,31,47)(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
8	7	cyclic	2	1	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
9	7	cyclic	3	1	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
10	7	cyclic	3	1	(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)
11	9	abelian	49	3,4,5	(8,36,22)(9,37,23)(10,38,24)(11,39,25)(12,40,26)(13,41,27)(14,42,28)(15,43,29)(16,44,30)(17,45,31)(18,46,32)(19,47,33)(20,48,34)(21,49,35)(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
12	14	dihedral	3	2,9	(2,36)(3,43)(4,8)(5,15)(6,22)(7,29)(9,39)(10,46)(12,18)(13,25)(14,32)(16,40)(17,47)(20,26)(21,33)(23,41)(24,48)(28,34)(30,42)(31,49)(38,44)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
13	14	cyclic	21	2,10	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,49,17,9,33,41,25)(2,47,20,11,29,42,24)(3,44,19,13,32,36,28)(4,43,21,10,30,40,27)(5,48,18,8,35,38,23)(6,46,15,14,31,37,26)(7,45,16,12,34,39,22)
14	18	solvable	49	6,7,11	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,22,6,8,4,36)(3,29,7,15,5,43)(9,25,41)(10,32,42,16,26,48)(11,39,37,23,27,13)(12,46,38,30,28,20)(14,18,40,44,24,34)(17,33,49)(19,47,45,31,35,21)
15	21	solvable	2	3,8	(2,6,4)(3,7,5)(9,13,11)(10,14,12)(16,20,18)(17,21,19)(23,27,25)(24,28,26)(30,34,32)(31,35,33)(37,41,39)(38,42,40)(44,48,46)(45,49,47)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
					(8,36,22)(9,37,23)(10,38,24)(11,39,25)(12,40,26)(13,41,27)(14,42,28)(15,43,29)(16,44,30)(17,45,31)(18,46,32)(19,47,33)(20,48,34)

16	21	cyclic	14	3,8	(21,49,35)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
17	21	solvable	14	4,8	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
18	21	solvable	14	5,8	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
19	21	solvable	21	4,9	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
20	21	solvable	21	4,10	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)
21	42	solvable	21	7,13,20	(1,49,17,9,33,41,25)(2,47,20,11,29,42,24)(3,44,19,13,32,36,28)(4,43,21,10,30,40,27)(5,48,18,8,35,38,23)(6,46,15,14,31,37,26)(7,45,16,12,34,39,22)(2,22,6,8,4,36)(3,29,7,15,5,43)(9,25,41)(10,32,42,16,26,48)(11,39,37,23,27,13)(12,46,38,30,28,20)(14,18,40,44,24,34)(17,33,49)(19,47,45,31,35,21)
22	42	solvable	21	7,12,19	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)(2,8,6,36,4,22)(3,15,7,43,5,29)(9,13,41,39,25,23)(10,20,42,46,26,30)(11,27,37)(12,34,38,18,28,44)(14,48,40,32,24,16)(17,21,49,47,33,31)(19,35,45)
23	49	abelian	1	8,9,10	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
					(2,4,6)(3,5,7)(9,11,13)(10,12,14)(16,18,20)

24	63	solvable	14	11,15,16,17, 18	(17,19,21)(23,25,27)(24,26,28)(30,32,34)(31, 33,35)(37,39,41)(38,40,42)(44,46,48)(45,47, 49)(1,7,3,2,5,6,4)(8,42,24,9,40,27,11,36,28, 10,37,26,13,39,22,14,38,23,12,41,25)(15,49, 31,16,47,34,18,43,35,17,44,33,20,46,29,21,4 5,30,19,48,32)
25	98	solvable	3	12,13,23	(1,28,38,30,12,20,46)(2,26,41,32,8,21,45)(3, 23,40,34,11,15,49)(4,22,42,31,9,19,48)(5,27, 39,29,14,17,44)(6,25,36,35,10,16,47)(7,24, 37,33,13,18,43)(1,33,49,41,17,25,9)(2,5,47, 48,20,18,11,8,29,35,42,38,24,23)(3,26,44,6, 19,46,13,15,32,14,36,31,28,37)(4,12,43,34, 21,39,10,22,30,7,40,45,27,16)
26	147	solvable	1	17,19,20,23	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11, 27,37)(12,28,38)(13,23,39)(14,24,40)(15,29, 43)(16,32,48)(17,33,49)(18,34,44)(19,35,45) (20,30,46)(21,31,47)(1,14,24,16,40,48,32)(2, 12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8, 28,17,37,47,34)(5,13,25,15,42,45,30)(6,11, 22,21,38,44,33)(7,10,23,19,41,46,29)(1,7,3,2, 5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20, 18)(22,28,24,23,26,27,25)(29,35,31,30,33,34, 32)(36,42,38,37,40,41,39)(43,49,45,44,47,48, 46)
27	147	solvable	1	18,23	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28) (11,41,23)(12,42,24)(13,37,25)(14,38,26) (15,43,29)(16,46,34)(17,47,35)(18,48,30) (19,49,31)(20,44,32)(21,45,33)(1,14,24,16, 40,48,32)(2,12,27,18,36,49,31)(3,9,26,20, 39,43,35)(4,8,28,17,37,47,34)(5,13,25,15, 42,45,30)(6,11,22,21,38,44,33)(7,10,23, 19,41,46,29)
28	147	solvable	2	15,16,23	(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21, 20,17,18,16)(22,26,28,27,24,25,23)(29,33,35, 34,31,32,30)(36,40,42,41,38,39,37)(43,47,49, 48,45,46,44)(1,8,22,15,36,43,29)(2,13,25,16, 41,46,30,6,11,23,20,39,44,34,4,9,27,18,37, 48,32)(3,14,26,17,42,47,31,7,12,24,21,40,45, 35,5,10,28,19,38,49,33)
29	294	solvable	1	6,25,27	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11, 41,23)(12,42,24)(13,37,25)(14,38,26)(15,43, 29)(16,46,34)(17,47,35)(18,48,30)(19,49,31) (20,44,32)(21,45,33)(1,43,49,21,17,10,9,30, 33,40,41,27,25,4)(2,29,47,42,20,24,11)(3,8, 44,35,19,38,13,23,32,5,36,48,28,18)(6,22,46, 7,15,45,14,16,31,12,37,34,26,39)
30	294	solvable	3	21,22,25,26	(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3, 45,17,10,31,38,24)(4,46,18,11,32,39,25)(5, 47,19,12,33,40,26)(6,48,20,13,34,41,27)(7, 49,21,14,35,42,28)(2,22,6,8,4,36)(3,29,7,15, 5,43)(9,25,41)(10,32,42,16,26,48)(11,39,37, 23,27,13)(12,46,38,30,28,20)(14,18,40,44,24, 34)(17,33,49)(19,47,45,31,35,21)
					(1,2,4,3,6,7,5)(8,37,25,10,41,28,12,36,23,11,

31	441	solvable	1	24,26,27,28	38,27,14,40,22,9,39,24,13,42,26)(15,44,32, 17,48,35,19,43,30,18,45,34,21,47,29,16,46, 31,20,49,33)(1,15,29,22,43,8,36)(2,20,32,23, 48,11,37,6,18,30,27,46,9,41,4,16,34,25,44, 13,39)(3,21,33,24,49,12,38,7,19,31,28,47,10, 42,5,17,35,26,45,14,40)
32	882	*G*	1	14,29,30,31	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11, 41,23)(12,42,24)(13,37,25)(14,38,26)(15,43, 29)(16,46,34)(17,47,35)(18,48,30)(19,49,31) (20,44,32)(21,45,33)(1,11,48,28,40,31)(2,18, 44,21,37,17)(3,4,46,49,42,38)(5,32,43,14,41, 24)(6,25,47,35,36,10)(7,39,45)(8,13,27,26, 33,29)(9,20,23,19,30,15)(12,34,22)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16, 19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33, 34,32)(36,42,38,37,40,41,39)(43,49,45,44,47, 48,46)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23) (3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47, 19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21, 14,35,42,28)
2	147	solvable	dihedral	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42) (11,27,37)(12,28,38)(13,23,39)(14,24,40) (15,29,43)(16,32,48)(17,33,49)(18,34,44) (19,35,45)(20,30,46)(21,31,47)(1,43,15, 8,29,36,22)(2,44,16,9,30,37,23)(3,45,17, 10,31,38,24)(4,46,18,11,32,39,25)(5,47, 19,12,33,40,26)(6,48,20,13,34,41,27)(7,49, 21,14,35,42,28)(1,7,3,2,5,6,4)(8,14,10,9,12, 13,11)(15,21,17,16,19,20,18)(22,28,24,23, 26,27,25)(29,35,31,30,33,34,32)(36,42,38, 37,40,41,39)(43,49,45,44,47,48,46)
3	147	solvable	cyclic	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28) (11,41,23)(12,42,24)(13,37,25)(14,38,26) (15,43,29)(16,46,34)(17,47,35)(18,48,30) (19,49,31)(20,44,32)(21,45,33)(1,39,13,47, 23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27, 33,16)(4,41,12,44,24,35,15)(5,37,10,49,22, 32,20)(6,40,9,45,28,29,18)(7,36,11,48,26, 30,17)
4	441	solvable	cyclic	1,2,3	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28) (11,41,23)(12,42,24)(13,37,25)(14,38,26) (15,43,29)(16,46,34)(17,47,35)(18,48,30) (19,49,31)(20,44,32)(21,45,33)(1,39,34,5, 37,31,7,36,32,6,40,30,3,42,29,4,41,33,2,38, 35)(8,11,13,12,9,10,14)(15,25,48,19,23,45, 21,22,46,20,26,44,17,28,43,18,27,47,16, 24,49)
5	294	solvable	cyclic	1,3	(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40) (11,23,41)(12,24,42)(13,25,37)(14,26,38) (15,29,43)(16,34,46)(17,35,47)(18,30,48) (19,31,49)(20,32,44)(21,33,45)(1,36,41,13, 9,44,49,28,25,32,33,19,17,3)(2,43,42,27,11,

					30,47,21,24,4,29,40,20,10)(5,15,38,6,8,37, 48,14,23,46,35,26,18,31)(7,22,39,34,12,16,45)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	2	21	cyclic	42	294	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
3	9	49	abelian	18	441	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
7	49	1	abelian	*G*	49	(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)

13. Let *G* be a primitive group of degree 49 with 5 generators. We have $|G| = 1176 = 2^3 \times 3 \times 7^2$.

Generators of *G*:	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49) (order 2)
	a_3 = (2,5)(3,6)(4,7)(9,12)(10,13)(11,14)(16,19)(17,20)(18,21)(23,26)(24,27)(25,28)(30,33)(31,34)(32,35)(37,40)(38,41)(39,42)(44,47)(45,48)(46,49) (order 2)
	a_4 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45) (order 3)
	a_5 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. *G* is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup *D* = [*G*, *G*] is a solvable group of order 294, generated by {(1,23,39,31,13,21,47)(2,25,38,34,14,19,43)(3,27,42,33,8,16,46)(4,24,41,35,12,15,44)(5,22,37,32,10,20,49)(6,28,40,29,9,18,45)(7,26,36,30,11,17,48)(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)} and $G/D \cong C_2^2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	294	solvable	abelian	C_2^2
2	147	solvable	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	294	solvable	abelian	C_2^2
2	49	abelian	solvable	$C_2 \times C_3$
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	588	solvable	1	(1,47,7,48,3,46,2,43,5,49,6,45,4,44)(8,40,14,41, 10,39,9,36,12,42,13,38,11,37)(15,26,21,27,17, 25,16,22,19,28,20,24,18,23)(29,33,35,34,31,32, 30)(1,47,37,6,49,38)(2,48,42,3,43,40)(4,46,39) (5,44,41,7,45,36)(8,26,30,13,28,31)(9,27,35,10, 22,33)(11,25,32)(12,23,34,14,24,29)(15,19,16, 20,21,17)
2	588	solvable	1	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23) (12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45) (26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,16, 41,26,35,46)(2,20,40,28,32,43)(3,17,38,24,31,45) (4,15,37,27,33,49)(5,21,39,22,30,48)(6,19,42,25, 29,44)(7,18,36,23,34,47)(8,9,13,12,14,11)
3	588	solvable	1	(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23, 41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16, 34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44) (21,33,45)(1,13,45,40)(2,27,46,19)(3,41,43,12) (4,20,44,26)(5,6,48,47)(7,34,49,33)(8,10,38,36) (9,24,39,15)(11,17,37,22)(14,31,42,29)(16,23,25, 18)(21,30,28,32)
4	392	solvable	3	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23) (12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45) (26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,12, 7,13,3,11,2,8,5,14,6,10,4,9)(15,47,21,48,17,46,16, 43,19,49,20,45,18,44)(22,33,28,34,24,32,23,29, 26,35,27,31,25,30)(36,40,42,41,38,39,37)
5	24	solvable	49	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23) (12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45) (26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(2,6,4) (3,7,5)(8,43,36,29,22,15)(9,48,39,30,27,18)(10,49, 40,31,28,19)(11,44,41,32,23,20)(12,45,42,33,24, 21)(13,46,37,34,25,

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21, 20,17,18,16)(22,26,28,27,24,25,23)(29,33,35, 34,31,32,30)(36,40,42,41,38,39,37)(43,47,49, 48,45,46,44)(1,43,15,8,29,36,22)(2,44,16,9, 30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32, 39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,

					27)(7,49,21,14,35,42,28)
2	98	solvable	dihedral	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
3	196	solvable	dihedral	1,2	(1,5)(2,7)(4,6)(8,33)(9,35)(10,31)(11,34)(12,29)(13,32)(14,30)(15,40)(16,42)(17,38)(18,41)(19,36)(20,39)(21,37)(22,47)(23,49)(24,45)(25,48)(26,43)(27,46)(28,44)(1,28,36,35,8,21,43,7,22,42,29,14,15,49)(2,27,37,34,9,20,44,6,23,41,30,13,16,48)(3,25,38,32,10,18,45,4,24,39,31,11,17,46)(5,26,40,33,12,19,47)
4	147	solvable	nilpotent	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,47,21,13,31,39,23)(2,43,19,14,34,38,25)(3,46,16,8,33,42,27)(4,44,15,12,35,41,24)(5,49,20,10,32,37,22)(6,45,18,9,29,40,28)(7,48,17,11,30,36,26)
5	294	solvable	abelian	1,2,4	(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)(1,47,21,13,31,39,23)(2,43,19,14,34,38,25)(3,46,16,8,33,42,27)(4,44,15,12,35,41,24)(5,49,20,10,32,37,22)(6,45,18,9,29,40,28)(7,48,17,11,30,36,26)
6	588	solvable	cyclic	1,2,4,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,36,37,2)(3,29,42,44)(4,15,40,9)(5,8,39,16)(6,22,41,23)(7,43,38,30)(10,32,21,47)(11,18,19,12)(13,25,20,26)(14,46,17,33)(24,34,28,48)(31,35,49,45)
7	588	solvable	cyclic	1,2,3,4,5	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,47,34,22,40,20)(2,44,30,23,37,16)(3,49,32,24,42,18)(4,45,35,25,38,21)(5,48,29,26,41,15)(6,43,33,27,36,19)(7,46,31,28,39,17)(8,12,13)(10,14,11)
8	588	solvable	cyclic	1,2,4,5	(2,29)(3,36)(4,43)(5,8)(6,15)(7,22)(9,33)(10,40)(11,47)(13,19)(14,26)(16,34)(17,41)(18,48)(21,27)(23,35)(24,42)(25,49)(31,37)(32,44)(39,45)(1,15,21,35,31,24,23,44,47,12,13,41,39,4)(2,43,19,14,34,38,25)(3,22,16,49,33,10,27,37,46,5,8,20,42,32)(6,36,18,7,29,17,28,30,45,26,9,48,40,11)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	147	nilpotent	8	G	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
3	3	49	cyclic	24	147	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
7	49	1	abelian	G	49	(1,22,36,29,8,15,43)(2,23,37,30,9,16,44)(3,24,38,31,10,17,45)(4,25,39,32,11,18,46)(5,26,40,33,12,19,47)(6,27,41,34,13,20,48)(7,28,42,35,14,21,49)(1,32,48,40,16,24,14)(2,31,49,36,18,27,12)(3,35,43,39,20,26,9)(4,34,47,37,17,28,8)(5,30,45,42,15,25,13)(6,33,44,38,21,22,11)(7,29,46,41,19,23,10)

14. Let G be a primitive group of degree 49 with 5 generators. We have $|G| = 1176 = 2^3\times3\times7^2$.

Generators of G:	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49) (order 2)
	a_3 = (2,5)(3,6)(4,7)(9,12)(10,13)(11,14)(16,19)(17,20)(18,21)(23,26)(24,27)(25,28)(30,33)(31,34)(32,35)(37,40)(38,41)(39,42)(44,47)(45,48)(46,49) (order 2)
	a_4 = (2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (order 3)
	a_5 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup D = [G, G] is a solvable group of order 98, generated by {(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)} and $G/D \cong C_2^2\times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	$C_2^2\times C_3$
2	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	$C_2^2 \times C_3$
2	49	abelian	nilpotent	C_2
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	588	solvable	1	(1,8,15)(2,14,20,5,11,17)(3,9,21,6,12,18)(4,10,16,7,13,19)(22,43,36)(23,49,41,26,46,38)(24,44,42,27,47,39)(25,45,37,28,48,40)(30,35,34,33,32,31)(1,3,5,4,7,2,6)(8,31,12,32,14,30,13,29,10,33,11,35,9,34)(15,38,19,39,21,37,20,36,17,40,18,42,16,41)(22,45,26,46,28,44,27,43,24,47,25,49,23,48)
2	588	solvable	1	(1,17,33,25,49,9,41)(2,38,29,18,47,23,42,8,20,5,24,35,11,48)(3,31,32,46,44,37,36,15,19,26,28,14,13,6)(4,45,30,39,43,16,40,22,21,12,27,7,10,34)(1,28,46,30,38,12)(2,24,47,29,42,11)(3,26,43,35,39,9)(4,23,45,33,36,14)(5,22,49,32,37,10)(6,27,48,34,41,13)(7,25,44,31,40,8)(15,21,18,16,17,19)
3	588	solvable	1	(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)(2,22,3,29,4,36,5,43,6,8,7,15)(9,28,17,30,25,38,33,46,41,12,49,20)(10,35,18,37,26,45,34,11,42,19,44,27)(13,14,21,16,23,24,31,32,39,40,47,48)
4	392	solvable	1	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,30,4,31,6,35,5,29,2,32,3,34,7,33)(8,44,11,45,13,49,12,43,9,46,10,48,14,47)(15,16,18,17,20,21,19)(22,37,25,38,27,42,26,36,23,39,24,41,28,40)
5	24	nilpotent	49	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(2,22,6,8,4,36)(3,29,7,15,5,43)(9,25,41)(10,32,42,16,26,48)(11,39,37,23,27,13)(12,46,38,30,28,20)(14,18,40,44,24,34)(17,33,49)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

2	147	solvable	nilpotent	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
3	98	solvable	abelian	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
4	294	solvable	abelian	1,2,3	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
5	196	solvable	cyclic	1,3	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)(7,35,49,42,21,28,14)
6	196	solvable	cyclic	1,3	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,9,4,10,6,14,5,8,2,11,3,13,7,12)(15,44,18,45,20,49,19,43,16,46,17,48,21,47)(22,30,25,31,27,35,26,29,23,32,24,34,28,33)(36,37,39,38,41,42,40)
					(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,

7	588	solvable	cyclic	1,2,3,4,5	20,27,13)(7,35,49,42,21,28,14)(2,43,3,8,4,15,5,22,6,29,7,36)(9,46,17,12,25,20,33,28,41,30,49,38)(10,11,18,19,26,27,34,35,42,37,44,45)(13,32,21,40,23,48,31,14,39,16,47,24)
8	588	solvable	cyclic	1,2,3,4,6	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,44,24,8,16,31)(2,45,22,9,17,29)(3,43,23,10,15,30)(4,49,27,11,21,34)(5,47,26,12,19,33)(6,46,28,13,18,35)(7,48,25,14,20,32)(36,37,38)(39,42,41)
9	196	solvable	cyclic	1,3	(1,49,17,9,33,41,25)(2,35,20,23,29,48,24,8,47,38,11,5,42,18)(3,14,19,37,32,6,28,15,44,31,13,26,36,46)(4,7,21,16,30,34,27,22,43,45,10,12,40,39)(1,28,38,30,12,20,46)(2,14,41,44,8,27,45,29,26,17,32,5,21,39)(3,35,40,16,11,6,49,36,23,10,34,47,15,25)(4,7,42,37,9,13,48,43,22,24,31,33,19,18)
10	588	solvable	cyclic	1,2,3,4,9	(1,30,46,38,20,28,12)(2,44,45,17,21,14,8,29,32,39,41,27,26,5)(3,16,49,10,15,35,11,36,34,25,40,6,23,47)(4,37,48,24,19,7,9,43,31,18,42,13,22,33)(1,9,17)(2,16,15,8,10,3)(4,44,20,22,14,38)(5,30,19,29,12,31)(6,23,21,36,11,45)(7,37,18,43,13,24)(25,49,41)(26,35,40,32,47,34)(27,28,42,39,46,48)
11	392	solvable	cyclic	1,3,5,6,9	(2,29)(3,36)(4,43)(5,8)(6,15)(7,22)(9,33)(10,40)(11,47)(13,19)(14,26)(16,34)(17,41)(18,48)(21,27)(23,35)(24,42)(25,49)(31,37)(32,44)(39,45)(1,33,7,34,3,32,2,29,5,35,6,31,4,30)(8,47,14,48,10,46,9,43,12,49,13,45,11,44)(15,19,21,20,17,18,16)(22,40,28,41,24,39,23,36,26,42,27,38,25,37)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	nilpotent	24	392	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
3	3	49	cyclic	24	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
7	49	1	abelian	*G*	49	(1,28,38,30,12,20,46)(2,26,41,32,8,21,45)(3,23,40,34,11,15,49)(4,22,42,31,9,19,48)(5,27,39,29,14,17,44)(6,25,36,35,10,16,47)(7,24,37,33,13,18,43)(1,49,17,9,33,41,25)

						(2,47,20,11,29,42,24)(3,44,19,13,32,36,28) (4,43,21,10,30,40,27)(5,48,18,8,35,38,23) (6,46,15,14,31,37,26)(7,45,16,12,34,39,22)

15. Let G be a primitive group of degree 49 with 2 generators. We have $|G| = 1176 = 2^3 \times 3 \times 7^2$.

Generators of G:	a_1 = (2,21,30,33,3,23,38,41,4,31,46,49,5,39,12,9,6,47,20,17,7,13,28,25) (8,40,35,42,15,48,37,44,22,14,45,10,29,16,11,18,36,24, 19,26,43,32,27,34) (order 24)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31) (4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35) (order 7)

G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is an abelian group of order 49, generated by {(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34, 24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)(1,26,42,34,10,18,44)(2,22,40,35,13,17,46) (3,25,37,29,12,21,48)(4,23,36,33,14,20,45)(5,28,41,31,11,16,43)(6,24,39,30,8,19,49)(7,27,38,32, 9,15,47)} and $G/D \cong C_3 \times C_8$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	$C_3 \times C_8$

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	$C_3 \times C_8$
2	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	588	solvable	1	(2,30,3,38,4,46,5,12,6,20,7,28)(8,35,15,37,22,45,29,11, 36,19,43,27)(9,47,17,13,25,21,33,23,41,31,49,39)(10,16, 18,24,26,32,34,40,42,48,44,14)(1,46,20,12,30,38,28)(2, 45,21,8,32,41,26)(3,49,15,11,34,40,23)(4,48,19,9,31,42, 22)(5,44,17,14,29,39,27)(6,47,16,10,35,36,25)(7,43,18, 13,33,37,24)
2	392	solvable	1	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30, 49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34, 22,24,35)(15,16,27,44,36,40,45,26)(1,16,32,24,48,14, 40)(2,18,31,27,49,12,36)(3,20,35,26,43,9,39)(4,17,34, 28,47,8,37)(5,15,30,25,45,13,42)(6,21,33,22,44,11,38) (7,19,29,23,46,10,41)
3	24	cyclic	49	(2,47,46,33,7,39,38,25,6,31,30,17,5,23,28,9,4,21,20,49, 3,13,12,41)(8,24,45,42,43,16,37,34,36,14,35,26,29,48, 27,18,22,40,19,10,15,32,11,44)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	cyclic	--	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46,32) (5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35)(1,40,14,48,24,32,16)

					(2,36,12,49,27,31,18)(3,39,9,43,26,35,20) (4,37,8,47,28,34,17)(5,42,13,45,25,30,15) (6,38,11,44,22,33,21)(7,41,10,46,23,29,19)
2	98	solvable	cyclic	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35) (12,30)(13,31)(14,32)(15,36)(16,40) (17,41)(18,42)(19,37)(20,38)(21,39) (22,43)(23,47)(24,48)(25,49)(26,44) (27,45)(28,46)(1,8,22,15,36,43,29) (2,9,23,16,37,44,30)(3,10,24,17,38,45,31) (4,11,25,18,39,46,32)(5,12,26,19,40,47,33) (6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (1,40,14,48,24,32,16)(2,36,12,49,27,31,18) (3,39,9,43,26,35,20)(4,37,8,47,28,34,17) (5,42,13,45,25,30,15)(6,38,11,44,22,33,21) (7,41,10,46,23,29,19)
3	196	solvable	cyclic	1,2	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37) (9,31,33,13)(10,48,34,24)(11,22,35,43) (14,42,32,18)(15,27,36,45)(16,44,40,26) (17,39,41,21)(23,25,47,49) (1,26,42,34,10,18,44)(2,22,40,35,13,17,46) (3,25,37,29,12,21,48)(4,23,36,33,14,20,45) (5,28,41,31,11,16,43)(6,24,39,30,8,19,49) (7,27,38,32,9,15,47)
4	392	solvable	cyclic	1,2,3	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13) (15,17,19,18,21,16,20)(22,24,26,25,28,23, 27)(29,31,33,32,35,30,34)(36,38,40,39,42, 37,41)(43,45,47,46,49,44,48)(1,4,24,13,19, 20,10,25)(2,12,43,28,21,22,47,9)(3,15,37, 36,17,5,35,33)(6,41,7,44,18,32,16,49)(8,34, 46,40,26,39,48,29)(11,14,38,30,27,23,31,42)
5	147	solvable	cyclic	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42) (11,27,37)(12,28,38)(13,23,39)(14,24,40) (15,29,43)(16,32,48)(17,33,49)(18,34,44) (19,35,45)(20,30,46)(21,31,47) (1,23,39,31,13,21,47)(2,25,38,34,14,19,43) (3,27,42,33,8,16,46)(4,24,41,35,12,15,44) (5,22,37,32,10,20,49)(6,28,40,29,9,18,45) (7,26,36,30,11,17,48)(1,16,32,24,48,14,40) (2,18,31,27,49,12,36)(3,20,35,26,43,9,39) (4,17,34,28,47,8,37)(5,15,30,25,45,13,42) (6,21,33,22,44,11,38)(7,19,29,23,46,10,41)
6	294	solvable	cyclic	1,2,5	(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33, 25,17)(10,44,42,34,26,18)(11,45,37,35,27, 19)(12,46,38,30,28,20)(13,47,39,31,23,21) (14,48,40,32,24,16)(1,21,31,23,47,13,39) (2,19,34,25,43,14,38)(3,16,33,27,46,8,42) (4,15,35,24,44,12,41)(5,20,32,22,49,10,37) (6,18,29,28,45,9,40)(7,17,30,26,48,11,36) (1,14,24,16,40,48,32)(2,12,27,18,36,49,31) (3,9,26,20,39,43,35)(4,8,28,17,37,47,34) (5,13,25,15,42,45,30)(6,11,22,21,38,44,33) (7,10,23,19,41,46,29)
					(2,30,3,38,4,46,5,12,6,20,7,28) (8,35,15,37,22,45,29,11,36,19,43,27)

7	588	solvable	cyclic	1,2,3,5,6	(9,47,17,13,25,21,33,23,41,31,49,39) (10,16,18,24,26,32,34,40,42,48,44,14) (1,45,19,11,35,37,27)(2,48,15,10,33,39,28) (3,47,18,14,30,41,22)(4,49,16,13,29,38,26) (5,46,21,9,34,36,24)(6,43,17,12,32,42,23) (7,44,20,8,31,40,25)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	cyclic	24	392	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)
3	3	49	cyclic	24	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
7	49	1	abelian	*G*	49	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)(1,31,47,39,21,23,13)(2,34,43,38,19,25,14)(3,33,46,42,16,27,8)(4,35,44,41,15,24,12)(5,32,49,37,20,22,10)(6,29,45,40,18,28,9)(7,30,48,36,17,26,11)

16. Let G be a primitive group of degree 49 with 3 generators. We have $|G| = 1176 = 2^3\times3\times7^2$.

Generators of *G*:	a_1 = (2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45) (order 4)
	a_2 = (2,30,3,38,4,46,5,12,6,20,7,28)(8,35,15,37,22,45,29,11,36,19,43,27)(9,47,17,13,25,21,33,23,41,31,49,39)(10,16,18,24,26,32,34,40,42,48,44,14) (order 12)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 98, generated by {(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)} and $G/D \cong C_2^2\times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	$C_2^2\times C_3$
2	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	$C_2^2 \times C_3$
2	49	abelian	nilpotent	C_2
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	588	solvable	1	(1,28,38,30,12,20,46)(2,26,41,32,8,21,45)(3,23,40,34,11,15,49)(4,22,42,31,9,19,48)(5,27,39,29,14,17,44)(6,25,36,35,10,16,47)(7,24,37,33,13,18,43)(2,30,3,38,4,46,5,12,6,20,7,28)(8,35,15,37,22,45,29,11,36,19,43,27)(9,47,17,13,25,21,33,23,41,31,49,39)(10,16,18,24,26,32,34,40,42,48,44,14)
2	588	solvable	1	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)(2,16,3,24,4,32,5,40,6,48,7,14)(8,31,15,39,22,47,29,13,36,21,43,23)(9,27,17,35,25,37,33,45,41,11,49,19)(10,12,18,20,26,28,34,30,42,38,44,46)
3	588	solvable	1	(1,18,34,26,44,10,42)(2,17,35,22,46,13,40)(3,21,29,25,48,12,37)(4,20,33,23,45,14,36)(5,16,31,28,43,11,41)(6,19,30,24,49,8,39)(7,15,32,27,47,9,38)(2,26,3,34,4,42,5,44,6,10,7,18)(8,9,15,17,22,25,29,33,36,41,43,49)(11,47,19,13,27,21,35,23,37,31,45,39)(12,32,20,40,28,48,30,14,38,16,46,24)
4	392	solvable	1	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,14,11,6)(2,32,9,22)(3,24,12,33)(4,40,8,38)(5,16,10,44)(7,48,13,21)(15,19,46,45)(17,41,47,42)(18,29,43,25)(20,23,49,30)(26,27,31,35)(28,36,34,39)
5	24	nilpotent	49	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(2,16,3,24,4,32,5,40,6,48,7,14)(8,31,15,39,22,47,29,13,36,21,43,23)(9,27,17,35,25,37,33,45,41,11,49,19)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,31,47,39,21,23,13)(2,34,43,38,19,25,14)(3,33,46,42,16,27,8)(4,35,44,41,15,24,12)(5,32,49,37,20,22,10)(6,29,45,40,18,28,9)(7,30,48,36,17,26,11)(1,43,15,8,29,36,22)

					(2,44,16,9,30,37,23)(3,45,17,10,31,38,24) (4,46,18,11,32,39,25)(5,47,19,12,33,40,26) (6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
2	147	solvable	nilpotent	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42) (11,27,37)(12,28,38)(13,23,39)(14,24,40) (15,29,43)(16,32,48)(17,33,49)(18,34,44) (19,35,45)(20,30,46)(21,31,47) (1,31,47,39,21,23,13)(2,34,43,38,19,25,14) (3,33,46,42,16,27,8)(4,35,44,41,15,24,12) (5,32,49,37,20,22,10)(6,29,45,40,18,28,9) (7,30,48,36,17,26,11)(1,43,15,8,29,36,22) (2,44,16,9,30,37,23)(3,45,17,10,31,38,24) (4,46,18,11,32,39,25)(5,47,19,12,33,40,26) (6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
3	98	solvable	abelian	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35) (12,30)(13,31)(14,32)(15,36)(16,40) (17,41)(18,42)(19,37)(20,38)(21,39) (22,43)(23,47)(24,48)(25,49)(26,44) (27,45)(28,46)(1,46,20,12,30,38,28) (2,45,21,8,32,41,26)(3,49,15,11,34,40,23) (4,48,19,9,31,42,22)(5,44,17,14,29,39,27) (6,47,16,10,35,36,25)(7,43,18,13,33,37,24) (1,8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46,32) (5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35)
4	196	solvable	cyclic	1,3	(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41, 29,17)(9,43,33,22)(11,31,35,13)(12,16, 30,40)(14,28,32,46)(15,49,36,25)(19, 39,37,21)(20,24,38,48)(23,27,47,45) (1,41,9,49,25,33,17)(2,42,11,47,24,29,20) (3,36,13,44,28,32,19)(4,40,10,43,27,30,21) (5,38,8,48,23,35,18)(6,37,14,46,26,31,15) (7,39,12,45,22,34,16)
5	196	solvable	cyclic	1,3	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19, 29,37)(9,31,33,13)(10,48,34,24)(11,22, 35,43)(14,42,32,18)(15,27,36,45)(16, 44,40,26)(17,39,41,21)(23,25,47,49) (1,19,35,27,45,11,37)(2,15,33,28,48,10,39) (3,18,30,22,47,14,41)(4,16,29,26,49,13,38) (5,21,34,24,46,9,36)(6,17,32,23,43,12,42) (7,20,31,25,44,8,40)
6	294	solvable	abelian	1,2,3	(1,46,20,12,30,38,28)(2,45,21,8,32,41,26) (3,49,15,11,34,40,23)(4,48,19,9,31,42,22) (5,44,17,14,29,39,27)(6,47,16,10,35,36,25) (7,43,18,13,33,37,24) (1,8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46,32) (5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35)(2,7,6,5,4,3) (8,43,36,29,22,15)(9,49,41,33,25,17) (10,44,42,34,26,18)(11,45,37,35,27,19) (12,46,38,30,28,20)(13,47,39,31,23,21) (14,48,40,32,24,16)

7	588	solvable	cyclic	1,2,3,4,6	(2,26,3,34,4,42,5,44,6,10,7,18) (8,9,15,17,22,25,29,33,36,41,43,49) (11,47,19,13,27,21,35,23,37,31,45,39) (12,32,20,40,28,48,30,14,38,16,46,24) (1,40,14,48,24,32,16)(2,36,12,49,27,31,18) (3,39,9,43,26,35,20)(4,37,8,47,28,34,17) (5,42,13,45,25,30,15)(6,38,11,44,22,33,21) (7,41,10,46,23,29,19)
8	588	solvable	cyclic	1,2,3,5,6	(2,30,3,38,4,46,5,12,6,20,7,28) (8,35,15,37,22,45,29,11,36,19,43,27) (9,47,17,13,25,21,33,23,41,31,49,39) (10,16,18,24,26,32,34,40,42,48,44,14) (1,32,48,40,16,24,14)(2,31,49,36,18,27,12) (3,35,43,39,20,26,9)(4,34,47,37,17,28,8) (5,30,45,42,15,25,13)(6,33,44,38,21,22,11) (7,29,46,41,19,23,10)
9	196	solvable	cyclic	1,3	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29, 39)(9,11,33,35)(10,38,34,20)(12,44,30,26) (13,22,31,43)(15,23,36,47)(17,19,41,37) (18,46,42,28)(25,27,49,45) (1,21,31,23,47,13,39)(2,19,34,25,43,14,38) (3,16,33,27,46,8,42)(4,15,35,24,44,12,41) (5,20,32,22,49,10,37)(6,18,29,28,45,9,40) (7,17,30,26,48,11,36)
10	392	solvable	cyclic	1,3,4,5,9	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29, 39)(9,11,33,35)(10,38,34,20)(12,44,30,26) (13,22,31,43)(15,23,36,47)(17,19,41,37) (18,46,42,28)(25,27,49,45) (1,19,26,29)(2,31,28,17)(3,48,24,39) (4,22,27,5)(6,9,25,14)(7,42,23,44) (8,35,12,16)(11,47,13,36)(15,45,33,38) (18,41,34,46)(20,21,32,30)(37,40,49,43)
11	588	solvable	cyclic	1,2,3,6,9	(2,16,3,24,4,32,5,40,6,48,7,14) (8,31,15,39,22,47,29,13,36,21,43,23) (9,27,17,35,25,37,33,45,41,11,49,19) (10,12,18,20,26,28,34,30,42,38,44,46) (1,38,12,46,28,30,20)(2,41,8,45,26,32,21) (3,40,11,49,23,34,15)(4,42,9,48,22,31,19) (5,39,14,44,27,29,17)(6,36,10,47,25,35,16) (7,37,13,43,24,33,18)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	nilpotent	24	392	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39) (9,11,33,35)(10,38,34,20)(12,44,30,26) (13,22,31,43)(15,23,36,47)(17,19,41,37) (18,46,42,28)(25,27,49,45)(2,20,5,38) (3,28,6,46)(4,30,7,12)(8,19,29,37) (9,31,33,13)(10,48,34,24)(11,22,35,43) (14,42,32,18)(15,27,36,45)(16,44,40,26) (17,39,41,21)(23,25,47,49)
3	3	49	cyclic	24	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42) (11,27,37)(12,28,38)(13,23,39)(14,24,40) (15,29,43)(16,32,48)(17,33,49)(18,34,44)

						(19,35,45)(20,30,46)(21,31,47)
7	49	1	abelian	*G*	49	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)(7,35,49,42,21,28,14)

17. Let *G* be a primitive group of degree 49 with 4 generators. We have $|G| = 1176 = 2^3\times3\times7^2$.

Generators of *G*:	a_1 = (2,29,24)(3,36,32)(4,43,40)(5,8,48)(6,15,14)(7,22,16)(9,41,25)(10,27,30)(11,20,42)(12,34,45)(17,49,33)(18,35,38)(19,28,44)(26,37,46) (order 3)
	a_2 = (2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46) (order 4)
	a_3 = (2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46) (order 4)
	a_4 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. *G* is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup *D* = [*G*, *G*] is a solvable group of order 392, generated by {(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(1,28,36,25)(2,14,40,18)(3,35,41,46)(4,7,42,39)(5,21,37,11)(6,49,38,32)(8,26,15,23)(9,12,19,16)(10,33,20,44)(13,47,17,30)(24,29,27,43)(31,34,48,45)} and $G/D \cong C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	cyclic	C_3

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	cyclic	C_3
2	98	solvable	solvable	C_2^2
3	49	abelian	solvable	C_2
4	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	392	solvable	1	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,10,31)(2,6,11,12)(3,21,8,49)(4,44,9,18)(5,38,13,36)(7,33,14,27)(15,37,45,39)(16,28,46,35)(17,47,43,20)(19,34,48,26)(23,29,32,24)(25,41,30,40)
				(2,5)(3,6)(4,7)(8,21,24,29,39,48)(9,20,27,33,38,45)

2	294	solvable	4	(10,18,26,34,42,44)(11,17,28,35,41,46)(12,19,25,30,37,49)(13,16,22,31,40,43)(14,15,23,32,36,47)(1,49,17,9,33,41,25)(2,47,20,11,29,42,24)(3,44,19,13,32,36,28)(4,43,21,10,30,40,27)(5,48,18,8,35,38,23)(6,46,15,14,31,37,26)(7,45,16,12,34,39,22)
3	24	solvable	49	(8,39,24)(9,38,27)(10,42,26)(11,41,28)(12,37,25)(13,40,22)(14,36,23)(15,47,32)(16,43,31)(17,46,35)(18,44,34)(19,49,30)(20,45,33)(21,48,29)(2,39,5,21)(3,47,6,23)(4,13,7,31)(8,48,29,24)(9,26,33,44)(10,17,34,41)(11,30,35,12)(14,36,32,15)(16,43,40,22)(18,25,42,49)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	392	solvable	cyclic	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,10,31)(2,6,11,12)(3,21,8,49)(4,44,9,18)(5,38,13,36)(7,33,14,27)(15,37,45,39)(16,28,46,35)(17,47,43,20)(19,34,48,26)(23,29,32,24)(25,41,30,40)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	nilpotent	24	392	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)
3	3	28	cyclic	42	*G*	(8,39,24)(9,38,27)(10,42,26)(11,41,28)(12,37,25)(13,40,22)(14,36,23)(15,47,32)(16,43,31)(17,46,35)(18,44,34)(19,49,30)(20,45,33)(21,48,29)
7	49	1	abelian	*G*	49	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16,21,18,19,17)(22,27,23,28,25,26,24)(29,34,30,35,32,33,31)(36,41,37,42,39,40,38)(43,48,44,49,46,47,45)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)

18. Let G be a primitive group of degree 49 with 4 generators. We have $|G| = 1176 = 2^3 \times 3 \times 7^2$.

	a_1 = (2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24)(7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39,23)(17,33,49)(20,26,27)(21,47,31)(28,34,35)(30,42,37)(38,44,45) (order 3)
	a_2 = (2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46) (order 4)
	a_3 = (2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)

Generators of G:	(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46) (order 4)
	a_4 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 392, generated by {(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(1,9,4,30)(3,44,5,23)(6,37,7,16)(8,11,32,29)(10,46,33,22)(12,25,31,43)(13,39,35,15)(14,18,34,36)(17,48,40,28)(19,27,38,49)(20,41,42,21)(24,45,47,26)} and $G/D \cong C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	cyclic	C_3

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	cyclic	C_3
2	98	solvable	solvable	C_2^2
3	49	abelian	solvable	C_2
4	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	392	solvable	1	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,10,31)(2,6,11,12)(3,21,8,49)(4,44,9,18)(5,38,13,36)(7,33,14,27)(15,37,45,39)(16,28,46,35)(17,47,43,20)(19,34,48,26)(23,29,32,24)(25,41,30,40)
2	294	solvable	4	(1,32,48,40,16,24,14)(2,31,49,36,18,27,12)(3,35,43,39,20,26,9)(4,34,47,37,17,28,8)(5,30,45,42,15,25,13)(6,33,44,38,21,22,11)(7,29,46,41,19,23,10)(2,36,40,5,15,16)(3,43,48,6,22,24)(4,8,14,7,29,32)(9,49,41,33,25,17)(10,35,46,34,11,28)(12,42,19,30,18,37)(13,21,23,31,39,47)(20,44,27,38,26,45)
3	24	solvable	49	(2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24)(7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39,23)(17,33,49)(20,26,27)(21,47,31)(28,34,35)(30,42,37)(38,44,45)(2,39,5,21)(3,47,6,23)(4,13,7,31)(8,48,29,24)(9,26,33,44)(10,17,34,41)(11,30,35,12)(14,36,32,15)(16,43,40,22)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
2	98	solvable	solvable	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
3	392	solvable	cyclic	1,2	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,10,31)(2,6,11,12)(3,21,8,49)(4,44,9,18)(5,38,13,36)(7,33,14,27)(15,37,45,39)(16,28,46,35)(17,47,43,20)(19,34,48,26)(23,29,32,24)(25,41,30,40)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	nilpotent	24	] 392	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)
3	3	196	cyclic	6	*G*	(2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24)(7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39,23)(17,33,49)(20,26,27)(21,47,31)(28,34,35)(30,42,37)(38,44,45)
7	49	1	abelian	*G*	49	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16,21,18,19,17)(22,27,23,28,25,26,24)(29,34,30,35,32,33,31)(36,41,37,42,39,40,38)(43,48,44,49,46,47,45)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)

19. Let G be a primitive group of degree 49 with 3 generators. We have $|G| = 1568 = 2^5 \times 7^2$.

Generators of G:	a_1 = (2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44) (order 2)
	a_2 = (2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44) (order 16)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 392, generated by {(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,28,38,30,12,20,46)(2,26,41,32,8,21,45)(3,23,40,34,11,15,49)(4,22,42,31,9,19,48)(5,27,39,29,14,17,44)(6,25,36,35,10,16,47)(7,24,37,33,13,18,43)} and $G/D \cong C_2^2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	abelian	C_2^2
2	196	solvable	nilpotent	C_2
3	98	solvable	nilpotent	C_2
4	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	abelian	C_2^2
2	49	abelian	nilpotent	C_8
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	784	solvable	1	(2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44)(1,10,26,18,42,44,34)(2,13,22,17,40,46,35)(3,12,25,21,37,48,29)(4,14,23,20,36,45,33)(5,11,28,16,41,43,31)(6,8,24,19,39,49,30)(7,9,27,15,38,47,32)
2	784	solvable	1	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(1,46,8,32,22,4,15,11,36,25,43,18,29,39)(2,37,9,44,23,30,16)(3,26,10,19,24,40,17,47,38,33,45,5,31,12)(6,14,13,28,27,21,20,42,41,49,48,35,34,7)
3	784	solvable	1	(2,11,5,35)(3,19,6,37)(4,27,7,45)(8,46,29,28)(9,34,33,10)(12,36,30,15)(13,24,31,48)(14,21,32,39)(16,23,40,47)(17,42,41,18)(20,43,38,22)(25,44,49,26)(1,5,8,41,44,46,37,13)(2,35,45,15,43,31,7,23)(3,38,32,12,49,14,33,39)(4,48,26,24,47,6,18,21)(9,17,27,42,36,28,20,10)(11,25,29,16,40,19,30,22)

4	32	nilpotent	49	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,46,20,12,30,38,28)(2,45,21,8,32,41,26)(3,49,15,11,34,40,23)(4,48,19,9,31,42,22)(5,44,17,14,29,39,27)(6,47,16,10,35,36,25)(7,43,18,13,33,37,24)
2	98	solvable	nilpotent	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
3	196	solvable	nilpotent	1,2	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,46,20,12,30,38,28)(2,45,21,8,32,41,26)(3,49,15,11,34,40,23)(4,48,19,9,31,42,22)(5,44,17,14,29,39,27)(6,47,16,10,35,36,25)(7,43,18,13,33,37,24)
4	392	solvable	abelian	1,2,3	(2,13,20,9,5,31,38,33)(3,21,28,17,6,39,46,41)(4,23,30,25,7,47,12,49)(8,32,19,18,29,14,37,42)(10,22,48,35,34,43,24,11)(15,40,27,26,36,16,45,44)(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)
5	784	solvable	cyclic	1,2,3,4	(2,13,20,9,5,31,38,33)(3,21,28,17,6,39,46,41)(4,23,30,25,7,47,12,49)(8,32,19,18,29,14,37,42)(10,22,48,35,34,43,24,11)(15,40,27,26,36,16,45,44)(1,14,11,6)(2,32,9,22)(3,24,12,33)(4,40,8,38)(5,16,10,44)(7,48,13,21)(15,19,46,45)(17,41,47,42)(18,29,43,25)(20,23,49,30)(26,27,31,35)(28,36,34,39)
6	784	solvable	cyclic	1,2,3,4	(2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44)(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)
					(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)

7	784	solvable	cyclic	1,2,3,4	(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(1,25,22,39,36,32,29,11,8,18,15,46,43,4)(2,9,23,16,37,44,30)(3,33,24,12,38,19,31,47,10,5,17,26,45,40)(6,49,27,7,41,28,34,42,13,35,20,14,48,21)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	32	49	nilpotent	32	*G*	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44)
7	49	1	abelian	*G*	49	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,20,30,28,46,12,38)(2,21,32,26,45,8,41)(3,15,34,23,49,11,40)(4,19,31,22,48,9,42)(5,17,29,27,44,14,39)(6,16,35,25,47,10,36)(7,18,33,24,43,13,37)

20. Let G be a primitive group of degree 49 with 5 generators. We have $|G| = 1764 = 2^2\times3^2\times7^2$.

Generators of *G*:	a_1 = (2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28) (order 4)
	a_2 = (2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46) (order 2)
	a_3 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45) (order 3)
	a_4 = (2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (order 3)
	a_5 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 147, generated by {(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)} and $G/D \cong C_3\times C_4$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	$C_3 \times C_4$

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	$C_3 \times C_4$
2	49	abelian	solvable	C_3
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	882	solvable	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,16,36,23,29,44)(2,15,37,22,30,43)(3,21,38,28,31,49)(4,19,39,26,32,47)(5,18,40,25,33,46)(6,20,41,27,34,48)(7,17,42,24,35,45)(8,9)(10,14)(11,12)
2	588	solvable	1	(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,26,16,11)(2,12,15,25)(3,33,21,39)(4,5,19,18)(6,47,20,46)(7,40,17,32)(8,22,23,9)(10,29,28,37)(13,43,27,44)(14,36,24,30)(31,35,42,38)(34,49,41,45)
3	588	solvable	3	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(2,43,3,8,4,15,5,22,6,29,7,36)(9,46,17,12,25,20,33,28,41,30,49,38)(10,11,18,19,26,27,34,35,42,37,44,45)(13,32,21,40,23,48,31,14,39,16,47,24)
4	36	solvable	49	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,43,3,8,4,15,5,22,6,29,7,36)(9,46,17,12,25,20,33,28,41,30,49,38)(10,11,18,19,26,27,34,35,42,37,44,45)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)

2	147	solvable	solvable	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28) (1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
3	98	solvable	solvable	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46) (1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28) (1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
4	294	solvable	dihedral	1,2,3	(1,4)(3,5)(6,7)(8,32)(9,30)(10,33)(11,29)(12,31)(13,35)(14,34)(15,39)(16,37)(17,40)(18,36)(19,38)(20,42)(21,41)(22,46)(23,44)(24,47)(25,43)(26,45)(27,49)(28,48) (1,4,5)(2,6,3)(8,25,40)(9,27,38)(10,23,41)(11,26,36)(12,22,39)(13,24,37)(14,28,42)(15,32,47)(16,34,45)(17,30,48)(18,33,43)(19,29,46)(20,31,44)(21,35,49) (1,42,18,44,10,26)(2,38,19,43,14,25)(3,40,15,49,11,23)(4,37,17,47,8,28)(5,36,21,46,9,24)(6,41,20,48,13,27)(7,39,16,45,12,22)(29,35,32,30,31,33)
5	147	solvable	cyclic	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33) (1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
6	441	solvable	cyclic	1,2,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33) (1,39,34,5,37,31,7,36,32,6,40,30,3,42,29,4,41,33,2,38,35)(8,11,13,12,9,10,14)(15,25,48,19,23,45,21,22,46,20,26,44,17,28,43,18,27,47,16,24,49)
					(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,

7	294	solvable	cyclic	1,3,5	35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)(1,16,32,24,48,14,40)(2,18,31,27,49,12,36)(3,20,35,26,43,9,39)(4,17,34,28,47,8,37)(5,15,30,25,45,13,42)(6,21,33,22,44,11,38)(7,19,29,23,46,10,41)
8	882	solvable	cyclic	1,2,3,4,5,6,7	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,16,43,37,8,30)(2,15,44,36,9,29)(3,21,45,42,10,35)(4,19,46,40,11,33)(5,18,47,39,12,32)(6,20,48,41,13,34)(7,17,49,38,14,31)(22,23)(24,28)(25,26)
9	588	solvable	cyclic	1,3,5,7	(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,13,42,16)(2,6,41,37)(3,48,39,30)(4,34,38,44)(5,27,40,23)(7,20,36,9)(8,14,21,15)(10,49,18,29)(11,35,17,43)(12,28,19,22)(24,47,25,33)(31,45,46,32)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	4	147	cyclic	12	588	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)
3	9	49	abelian	36	441	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
7	49	1	abelian	*G*	49	(1,36,8,43,22,29,15)(2,37,9,44,23,30,16)(3,38,10,45,24,31,17)(4,39,11,46,25,32,18)(5,40,12,47,26,33,19)(6,41,13,48,27,34,20)(7,42,14,49,28,35,21)(1,48,16,14,32,40,24)(2,49,18,12,31,36,27)(3,43,20,9,35,39,26)(4,47,17,8,34,37,28)(5,45,15,13,30,42,25)(6,44,21,11,33,38,22)(7,46,19,10,29,41,23)

21. Let *G* be a primitive group of degree 49 with 5 generators. We have $|G| = 1764 = 2^2 \times 3^2 \times 7^2$.

	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46) (order 2)

Generators of G:	a_3 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45) (order 3)
	a_4 = (2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (order 3)
	a_5 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup D = [G, G] is a solvable group of order 147, generated by {(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)} and $G/D \cong C_2^2 \times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	abelian	$C_2^2 \times C_3$

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	abelian	$C_2^2 \times C_3$
2	49	abelian	solvable	C_3
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	882	solvable	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,14,29,21,22,42)(2,13,30,20,23,41)(3,11,31,18,24,39)(4,10,32,17,25,38)(5,12,33,19,26,40)(6,9,34,16,27,37)(7,8,35,15,28,36)(43,49)(44,48)(45,46)
2	882	solvable	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,43,45,38,37,2)(3,36,44)(4,22,48,31,42,9)(5,15,47,17,40,16)(6,29,49,10,39,23)(7,8,46,24,41,30)(11,25,27,34,35,14)(12,18,26,20,33,21)(13,32,28)
3	882	solvable	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,22,24,17,16,2)(3,15,23)(4,43,27,10,21,30)(5,36,26,38,19,37)(6,8,28,31,18,44)(7,29,25,45,20,9)(11,49,34)(12,42,33,39,47,41)(13,14,35,32,46,48)
4	588	solvable	1	(2,3,4,5,6,7)(8,43,36,29,22,15)(9,45,39,33,27,21)(10,46,40,34,28,16)(11,47,41,35,23,17)(12,48,42,30,24,18)(13,49,37,31,25,19)(14,44,38,32,26,20)

				(1,23,17,48,33,8,25,38,49,5,9,18,41,35)(2,16,20,34,29,22,24,45,47,12,11,39,42,7)(3,44,19,13,32,36,28)(4,37,21,6,30,15,27,31,43,26,10,46,40,14)
5	588	solvable	3	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,43,45,38,37,2)(3,36,44)(4,22,48,31,42,9)(5,15,47,17,40,16)(6,29,49,10,39,23)(7,8,46,24,41,30)(11,25,27,34,35,14)(12,18,26,20,33,21)(13,32,28)
6	36	solvable	49	(2,22,6,8,4,36)(3,29,7,15,5,43)(9,25,41)(10,32,42,16,26,48)(11,39,37,23,27,13)(12,46,38,30,28,20)(14,18,40,44,24,34)(17,33,49)(19,47,45,31,35,21)(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
2	147	solvable	dihedral	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
3	98	solvable	solvable	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
					(1,4,2,7,3,6)(8,18,23,35,38,48)(9,21,24,34,36,46)(10,20,22,32,37,49)(11,16,28,31,41,

4	294	solvable	dihedral	1,2,3	43)(12,19,26,33,40,47)(13,15,25,30,42,45)(14,17,27,29,39,44)(1,36,43)(2,39,48)(3,40,49)(4,41,44)(5,42,45)(6,37,46)(7,38,47)(8,29,22)(9,32,27)(10,33,28)(11,34,23)(12,35,24)(13,30,25)(14,31,26)(16,18,20)(17,19,21)(1,42,18,44,10,26)(2,38,19,43,14,25)(3,40,15,49,11,23)(4,37,17,47,8,28)(5,36,21,46,9,24)(6,41,20,48,13,27)(7,39,16,45,12,22)(29,35,32,30,31,33)
5	147	solvable	abelian	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
6	441	solvable	abelian	1,2,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,39,34,5,37,31,7,36,32,6,40,30,3,42,29,4,41,33,2,38,35)(8,11,13,12,9,10,14)(15,25,48,19,23,45,21,22,46,20,26,44,17,28,43,18,27,47,16,24,49)
7	294	solvable	cyclic	1,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,43,46,18,20,13,12,33,30,37,38,24,28,7)(2,36,45,25,21,6,8,47,32,16,41,10,26,35)(3,22,49,4,15,48,11,19,34,9,40,31,23,42)(5,29,44,39,17,27,14)
8	294	solvable	cyclic	1,3,5	(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
9	882	solvable	cyclic	1,2,5,6,7	(1,7,42,38,10,9,44,47,26,27,34,32,18,15)(2,49,40,24,13,30,46,19,22,6,35,39,17,8)(3,14,37,45,12,23,48,33,25,20,29,4,21,36)(5,28,41,31,11,16,43)(1,18,31,41,23,14)(2,11,29,20,24,42)(3,39,30,13,22,21)(4,32,34,27,28,7)(5,46,33,48,26,49)(6,25,35)(8,15,17,38,37,9)(10,36,16)(12,43,19,45,40,44)(1,23,17)(2,24,15)(3,22,16)(4,28,20)(5,26,19)(6,25,21)(7,27,18)(8,30,45)(9,31,43)(10,29,44)(11,35,48)(12,33,47)(13,32,49)(14,34,46)(36,37,38)(39,42,41)
					(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)

10	882	solvable	cyclic	1,2,3,4,5,6,8	(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,39,22,18,29,11)(2,37,23,16,30,9)(3,40,24,19,31,12)(4,36,25,15,32,8)(5,38,26,17,33,10)(6,42,27,21,34,14)(7,41,28,20,35,13)(43,46)(45,47)(48,49)
11	294	solvable	cyclic	1,5	(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,36,41,13,9,44,49,28,25,32,33,19,17,3)(2,43,42,27,11,30,47,21,24,4,29,40,20,10)(5,15,38,6,8,37,48,14,23,46,35,26,18,31)(7,22,39,34,12,16,45)
12	882	solvable	cyclic	1,2,5,6	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,8,11,46,45,3)(2,15,12,32,49,38)(4,43,10)(5,29,14,39,44,17)(6,22,13,25,48,24)(7,36,9,18,47,31)(16,19,33,35,42,37)(20,26,34,28,41,23)(21,40,30)
13	588	solvable	cyclic	--	(2,3,4,5,6,7)(8,43,36,29,22,15)(9,45,39,33,27,21)(10,46,40,34,28,16)(11,47,41,35,23,17)(12,48,42,30,24,18)(13,49,37,31,25,19)(14,44,38,32,26,20)(1,31,25,14,41,15,33,46,9,6,17,26,49,37)(2,3,24,28,42,36,29,32,11,13,20,19,47,44)(4,10,27,21,40,43,30)(5,45,23,7,38,22,35,39,8,34,18,12,48,16)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	4	147	abelian	12	588	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
3	9	49	abelian	36	441	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
7	49	1	abelian	*G*	49	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16,21,18,19,17)(22,27,23,28,25,26,24)(29,34,30,35,32,33,31)(36,41,37,42,39,40,38)(43,48,44,49,46,47,45)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)

					(4,46,18,11,32,39,25)(5,47,19,12,33,40,26) (6,48,20,13,34,41,27)(7,49,21,14,35,42,28)

22. Let G be a primitive group of degree 49 with 3 generators. We have $|G| = 2352 = 2^4\times3\times7^2$.

Generators of G:	a_1 = (2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45) (order 4)
	a_2 = (2,21,30,33,3,23,38,41,4,31,46,49,5,39,12,9,6,47,20,17,7,13,28,25)(8,40,35,42,15,48,37,44,22,14,45,10,29,16,11,18,36,24,19,26,43,32,27,34) (order 24)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup D = [G, G] is a solvable group of order 196, generated by {(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,28,38,30,12,20,46)(2,26,41,32,8,21,45)(3,23,40,34,11,15,49)(4,22,42,31,9,19,48)(5,27,39,29,14,17,44)(6,25,36,35,10,16,47)(7,24,37,33,13,18,43)} and $G/D \cong C_2^2\times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	$C_2^2\times C_3$
2	98	solvable	nilpotent	C_2
3	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	$C_2^2\times C_3$
2	49	abelian	nilpotent	C_4
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	1176	solvable	1	(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31)(2,47,46,33,7,39,38,25,6,31,30,17,5,23,28,9,4,21,20,49,3,13,12,41)(8,24,45,42,43,16,37,34,36,14,35,26,29,48,27,18,22,40,19,10,15,32,11,44)
2	1176	solvable	1	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(1,14,23,46,13,15,47,37,21,6,39,26)(2,5,27,28,12,11,49,43,18,16,36,41)(3,44,24,20,10,40,45,7,17,25,38,8)(4,29,22,30,9,34,48,33,19,35,42,32)
				(2,43,5,22)(3,8,6,29)(4,15,7,36)(9,48,33,24)(10,13,34,31)(11,20,35,38)(12,27,30,45)(14,

3	1176	solvable	1	41,32,17)(16,49,40,25)(18,21,42,39)(19,28,37,46)(23,44,47,26)(1,41,15,14,43,30,36,3,8,18,29,47)(2,21,17,44,46,38,40,11,13,33,35,6)(4,26,19,27,48,28,42,23,9,24,31,25)(5,45,20,39,49,12,37,34,10,7,32,16)
4	784	solvable	1	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,11,2,33)(3,22,7,44)(4,38,5,21)(8,36,30,16)(9,27,29,48)(10,49,35,24)(12,31,32,14)(13,18,34,40)(15,17,37,42)(19,47,39,25)(20,28,41,45)(23,46,43,26)
5	48	nilpotent	49	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(2,16,3,24,4,32,5,40,6,48,7,14)(8,31,15,39,22,47,29,13,36,21,43,23)(9,27,17,35,25,37,33,45,41,11,49,19)(10,12,18,20,26,28,34,30,42,

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
2	147	solvable	nilpotent	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
3	98	solvable	nilpotent	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
					(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)

4	294	solvable	nilpotent	1,2,3	(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
5	196	solvable	abelian	1,3	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,46,20,12,30,38,28)(2,45,21,8,32,41,26)(3,49,15,11,34,40,23)(4,48,19,9,31,42,22)(5,44,17,14,29,39,27)(6,47,16,10,35,36,25)(7,43,18,13,33,37,24)
6	392	solvable	cyclic	1,3,5	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,19,17,6)(2,37,18,32)(3,35,15,42)(4,45,16,43)(5,27,20,12)(7,11,21,23)(8,9,24,25)(10,36,22,31)(13,46,26,44)(14,34,28,40)(29,48,38,47)(30,33,39,41)
7	392	solvable	cyclic	1,3,5	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
8	588	solvable	abelian	1,2,3,4,5	(1,46,20,12,30,38,28)(2,45,21,8,32,41,26)(3,49,15,11,34,40,23)(4,48,19,9,31,42,22)(5,44,17,14,29,39,27)(6,47,16,10,35,36,25)(7,43,18,13,33,37,24)(2,12,3,20,4,28,5,30,6,38,7,46)(8,11,15,19,22,27,29,35,36,37,43,45)(9,23,17,31,25,39,33,47,41,13,49,21)(10,40,18,48,26,14,34,16,42,24,44,32)
9	1176	solvable	cyclic	1,2,3,4,5,6,8	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,26,15,32,36,45,22,2,29,21,43,41)(3,4,16,17,42,37,27,28,33,34,46,47)(5,49,18,6,38,19,23,39,35,24,48,30)(7,13,20,12,40,11,25,10,31,9,44,14)
10	1176	solvable	cyclic	1,2,3,4,5,7,8	(2,47,46,33,7,39,38,25,6,31,30,17,5,23,28,9,4,21,20,49,3,13,12,41)(8,24,45,42,43,16,37,34,36,14,35,26,29,48,27,18,22,40,19,10,15,32,11,44)(1,35,45,37,19,27,11)(2,33,48,39,15,28,10)(3,30,47,41,18,22,14)(4,29,49,38,16,26,13)(5,34,46,36,21,24,9)(6,32,43,42,17,23,12)(7,31,44,40,20,25,8)
					(2,34,5,10)(3,42,6,18)(4,44,7,26)(8,17,29,41)(9,22,33,43)(11,13,35,31)(12,40,30,16)(14,46,32,28)(15,25,36,49)(19,21,37,39)

11	392	solvable	cyclic	1,3	(20,48,38,24)(23,45,47,27)(1,31,38,43)(2,36,39,3)(4,27,37,26)(5,12,41,20)(6,49,40,35)(7,18,42,9)(8,23,17,25)(10,19,15,13)(11,46,16,30)(14,29,21,45)(22,47,24,34)(32,33,44,48)
12	784	solvable	cyclic	--	(2,34,5,10)(3,42,6,18)(4,44,7,26)(8,17,29,41)(9,22,33,43)(11,13,35,31)(12,40,30,16)(14,46,32,28)(15,25,36,49)(19,21,37,39)(20,48,38,24)(23,45,47,27)(1,39,29,20,16,33,37,6)(2,21,27,24,15,3,13,14)(4,23,44,11,19,8,43,26)(5,47,17,42,18,46,7,31)(9,38,45,30,22,35,49,36)(10,25,41,40,28,12,34,32)
13	1176	solvable	cyclic	--	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,5,28,25,12,13,46,45,20,15,38,42)(2,43,23,21,9,40,44,4,16,27,37,10)(3,11,22,48,14,17,47,36,18,7,41,26)(6,31,24,29,8,35,49,33,19,32,39,34)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	16	49	nilpotent	48	784	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)
3	3	49	cyclic	48	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
7	49	1	abelian	*G*	49	(1,35,45,37,19,27,11)(2,33,48,39,15,28,10)(3,30,47,41,18,22,14)(4,29,49,38,16,26,13)(5,34,46,36,21,24,9)(6,32,43,42,17,23,12)(7,31,44,40,20,25,8)(1,36,8,43,22,29,15)(2,37,9,44,23,30,16)(3,38,10,45,24,31,17)(4,39,11,46,25,32,18)(5,40,12,47,26,33,19)(6,41,13,48,27,34,20)(7,42,14,49,28,35,21)

23. Let *G* be a primitive group of degree 49 with 2 generators. We have $|G| = 2352 = 2^4 \times 3 \times 7^2$.

Generators of *G*:	a_1 = (2,40,21,35,30,42,33,15,3,48,23,37,38,44,41,22,4,14,31,45,46,10,49,29,5,16,39,11,12,18,9,36,6,24,47,19,20,26,17,43,7,32,13,27,28,34,25,8) (order 48)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

G is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial. The derived subgroup *D* = [*G*, *G*] is an abelian group of order 49, generated by {(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39, 17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)(1,42,10,44,26,34,18)(2,40,13,46,22,35,17)(3,37,12,48,25,29,21)(4,36,14,45,23,33,20)(5,41,11,43,28,31,16)(6,39,8,49,24,30,19)(7,38,9,47,27,32,15)} and $G/D \cong C_3{\times}C_{16}$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	$C_3{\times}C_{16}$

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	$C_3{\times}C_{16}$
2	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	1176	solvable	1	(2,47,46,33,7,39,38,25,6,31,30,17,5,23,28,9,4,21,20,49,3,13,12,41)(8,24,45,42,43,16,37,34,36,14,35,26,29,48,27,18,22,40,19,10,15,32,11,44)(1,48,16,14,32,40,24)(2,49,18,12,31,36,27)(3,43,20,9,35,39,26)(4,47,17,8,34,37,28)(5,45,15,13,30,42,25)(6,44,21,11,33,38,22)(7,46,19,10,29,41,23)
2	784	solvable	1	(2,24,31,35,20,10,33,43,5,48,13,11,38,34,9,22)(3,32,39,37,28,18,41,8,6,14,21,19,46,42,17,29)(4,40,47,45,30,26,49,15,7,16,23,27,12,44,25,36)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
3	48	cyclic	49	(2,40,21,35,30,42,33,15,3,48,23,37,38,44,41,22,4,14,31,45,46,10,49,29,5,16,39,11,12,18,9,36,6,24,47,19,20,26,17,43,7,32,13,27,28,34,25,8)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	cyclic	--	(1,24,40,32,14,16,48)(2,27,36,31,12,18,49)(3,26,39,35,9,20,43)(4,28,37,34,8,17,47)(5,25,42,30,13,15,45)(6,22,38,33,11,21,44)(7,23,41,29,10,19,46)(1,47,21,13,31,39,23)(2,43,19,14,34,38,25)(3,46,16,8,33,42,27)(4,44,15,12,35,41,24)(5,49,20,10,32,37,22)(6,45,18,9,29,40,28)(7,48,17,11,30,36,26)
2	98	solvable	cyclic	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,24,40,32,14,16,48)(2,27,36,31,12,18,49)(3,26,39,35,9,20,43)(4,28,37,34,8,17,47)(5,25,42,30,13,15,45)(6,22,38,33,11,21,44)(7,23,41,29,10,19,46)(1,47,21,13,31,39,23)(2,43,19,14,34,38,25)(3,46,16,8,33,42,27)(4,44,15,12,35,41,24)(5,49,20,10,32,37,22)(6,45,18,9,29,40,28)(7,48,17,11,30,36,26)
					(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)

3	196	solvable	cyclic	1,2	(23,25,47,49)(1,49,17,9,33,41,25)(2,47,20,11,29,42,24)(3,44,19,13,32,36,28)(4,43,21,10,30,40,27)(5,48,18,8,35,38,23)(6,46,15,14,31,37,26)(7,45,16,12,34,39,22)
4	392	solvable	cyclic	1,2,3	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)
5	784	solvable	cyclic	1,2,3,4	(1,27,42,5,6,20,11,16,25,7,48,24,28,35,8,30)(2,41,47,31,36,40,26,14,23,49,38,19,46,45,3,13)(4,34,29,43,10,44,17,32,22,21,18,39,12,37,33,15)(1,35,46,18,49,29,3,24)(2,4,12,38,48,45,40,11)(5,44,28,23,47,6,15,20)(7,41,30,8,43,9,34,42)(10,14,19,22,39,36,26,21)(13,25,31,16,37,17,32,27)
6	147	solvable	cyclic	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,23,39,31,13,21,47)(2,25,38,34,14,19,43)(3,27,42,33,8,16,46)(4,24,41,35,12,15,44)(5,22,37,32,10,20,49)(6,28,40,29,9,18,45)(7,26,36,30,11,17,48)
7	294	solvable	cyclic	1,2,6	(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16,21,18,19,17)(22,27,23,28,25,26,24)(29,34,30,35,32,33,31)(36,41,37,42,39,40,38)(43,48,44,49,46,47,45)(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)
8	588	solvable	cyclic	1,2,3,6,7	(2,30,3,38,4,46,5,12,6,20,7,28)(8,35,15,37,22,45,29,11,36,19,43,27)(9,47,17,13,25,21,33,23,41,31,49,39)(10,16,18,24,26,32,34,40,42,48,44,14)(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)
9	1176	solvable	cyclic	1,2,3,4,6,7,8	(2,47,46,33,7,39,38,25,6,31,30,17,5,23,28,9,4,21,20,49,3,13,12,41)(8,24,45,42,43,16,37,34,36,14,35,26,29,48,27,18,22,40,19,10,15,32,11,44)(1,42,10,44,26,34,18)(2,40,13,46,22,35,17)(3,37,12,48,25,29,21)(4,36,14,45,23,33,20)(5,41,11,43,28,31,16)(6,39,8,49,24,30,19)(7,38,9,47,27,32,15)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	generators
2	16	49	cyclic	48	784	(2,24,31,35,20,10,33,43,5,48,13,11,38,34,9,22) (3,32,39,37,28,18,41,8,6,14,21,19,46,42,17,29) (4,40,47,45,30,26,49,15,7,16,23,27,12,44,25,36)
3	3	49	cyclic	48	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42) (11,27,37)(12,28,38)(13,23,39)(14,24,40) (15,29,43)(16,32,48)(17,33,49)(18,34,44) (19,35,45)(20,30,46)(21,31,47)
7	49	1	abelian	G	49	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37) (3,17,31,24,45,10,38)(4,18,32,25,46,11,39) (5,19,33,26,47,12,40)(6,20,34,27,48,13,41) (7,21,35,28,49,14,42)(1,49,17,9,33,41,25) (2,47,20,11,29,42,24)(3,44,19,13,32,36,28) (4,43,21,10,30,40,27)(5,48,18,8,35,38,23) (6,46,15,14,31,37,26)(7,45,16,12,34,39,22)

24. Let G be a primitive group of degree 49 with 3 generators. We have $|G| = 2352 = 2^4\times3\times7^2$.

Generators of G:	a_1 = (2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44) (order 2)
	a_2 = (2,21,30,33,3,23,38,41,4,31,46,49,5,39,12,9,6,47,20,17,7,13,28,25)(8,40,35,42,15,48,37,44,22,14,45,10,29,16,11,18,36,24,19,26,43,32,27,34) (order 24)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup D = [G, G] is a solvable group of order 196, generated by {(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)} and $G/D \cong C_2^2\times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	$C_2^2\times C_3$
2	98	solvable	nilpotent	C_2
3	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	$C_2^2\times C_3$
2	49	abelian	nilpotent	C_4
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
				(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29) (15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)

1	1176	solvable	1	(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(1,33,42,11,26,6,18,38,34,22,10,21)(2,7,37,40,23,25,16,20,30,31,9,8)(3,24,36,15,28,35,19,12,32,4,13,41)(5,43,39,49,27,47,17,46,29,48,14,45)
2	1176	solvable	1	(2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44)(1,14,4,17,2,27,7,29,3,39,6,44)(8,22,18,32,23,37,35,49,38,10,48,20)(9,19,21,26,24,33,34,40,36,47,46,12)(11,42,16,45,28,13,31,15,41,25,43,30)
3	1176	solvable	1	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16,21,18,19,17)(22,27,23,28,25,26,24)(29,34,30,35,32,33,31)(36,41,37,42,39,40,38)(43,48,44,49,46,47,45)(2,49,28,31,7,41,20,23,6,33,12,21,5,25,46,13,4,17,38,47,3,9,30,39)(8,10,27,14,43,44,19,48,36,42,11,40,29,34,45,32,22,26,37,24,15,18,35,16)
4	784	solvable	1	(2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44)(1,19,6,17,2,15,7,20,4,16,5,21,3,18)(8,23,13,28,9,25,14,26,11,24,12,22,10,27)(29,41,34,37,30,42,35,39,32,40,33,38,31,36)(43,46,48,47,44,45,49)
5	48	nilpotent	49	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(2,49,6,33,4,17)(3,9,7,41,5,25)(8,22,36)(10,16,42,48,26,32)(11,45,37,35,27,19)(12,39,38,23,28,13)(14,34,40,18,24,44)(15,29,43)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
2	147	solvable	nilpotent	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
					(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,15,

3	98	solvable	nilpotent	1	29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
4	294	solvable	nilpotent	1,2,3	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
5	196	solvable	abelian	1,3	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)
6	588	solvable	abelian	1,2,3,4,5	(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)(2,30,3,38,4,46,5,12,6,20,7,28)(8,35,15,37,22,45,29,11,36,19,43,27)(9,47,17,13,25,21,33,23,41,31,49,39)(10,16,18,24,26,32,34,40,42,48,44,14)
7	392	solvable	cyclic	1,3,5	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)
8	392	solvable	cyclic	1,3,5	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,12)(2,8)(3,11)(4,9)(5,14)(6,10)(7,13)(15,49)(16,47)(17,44)(18,43)(19,48)(20,46)(21,45)(22,31)(23,34)(24,33)(25,35)(26,32)(27,29)(28,30)
9	1176	solvable	cyclic	1,2,3,4,5,6,7	(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)(2,47,46,33,7,39,38,

					25,6,31,30,17,5,23,28,9,4,21,20,49,3,13,12,41)(8,24,45,42,43,16,37,34,36,14,35,26,29,48,27,18,22,40,19,10,15,32,11,44)
10	1176	solvable	cyclic	1,2,3,4,5,6,8	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,12,30,20,28,38)(2,9,35,21,22,36)(3,14,33,15,27,37)(4,10,32,19,25,41)(5,13,34,17,24,40)(6,8,31,16,26,42)(7,11,29,18,23,39)(43,49,44)(45,48,47)
11	392	solvable	cyclic	1,3	(2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44)(1,11,32,6,48,33,40,44,16,38,24,21,14,22)(2,26,31,9,49,3,36,35,18,43,27,39,12,20)(4,17,34,28,47,8,37)(5,7,30,29,45,46,42,41,15,19,25,23,13,10)
12	1176	solvable	cyclic	--	(2,13)(3,21)(4,23)(5,31)(6,39)(7,47)(8,19)(9,38)(10,34)(11,43)(12,25)(15,27)(17,46)(18,42)(20,33)(22,35)(26,44)(28,41)(29,37)(30,49)(36,45)(1,40,2,27,7,10)(3,43,5,30,6,21)(8,33,37,20,28,45)(9,41,42,24,22,12)(11,25,39)(13,49,38,29,26,16)(14,17,36,47,23,34)(15,19,44,48,35,31)(18,32,46)
13	784	solvable	cyclic	--	(2,13)(3,21)(4,23)(5,31)(6,39)(7,47)(8,19)(9,38)(10,34)(11,43)(12,25)(15,27)(17,46)(18,42)(20,33)(22,35)(26,44)(28,41)(29,37)(30,49)(36,45)(1,10,2,13,4,14,3,12,6,8,7,9,5,11)(15,46,16,45,18,48,17,49,20,47,21,43,19,44)(22,29,23,30,25,32,24,31,27,34,28,35,26,33)(36,41,37,42,39,40,38)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	16	49	nilpotent	48	784	(2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44)(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)
3	3	49	cyclic	48	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
7	49	1	abelian	*G*	49	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,38,12,46,28,30,20)(2,41,8,45,26,32,21)(3,40,11,49,23,34,15)(4,42,9,48,22,31,19)(5,39,14,44,27,29,17)(6,36,10,47,25,35,16)(7,37,13,43,24,33,18)

25. Let G be a primitive group of degree 49 with 5 generators. We have $|G| = 2352 = 2^4\times3\times7^2$.

Generators of G:	a_1 = (2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25) (order 4)
	a_2 = (2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24)(7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39,23)(17,33,49)(20,26,27)(21,47,31)(28,34,35)(30,42,37)(38,44,45) (order 3)
	a_3 = (2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46) (order 4)
	a_4 = (2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46) (order 4)
	a_5 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup D = [G, G] is a solvable group of order 1176, generated by {(2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46)(1,14,13)(2,25,5)(3,38,45)(4,19,30)(6,43,42)(7,34,15)(8,21,48)(9,39,40)(10,31,24)(11,47,16)(12,23,32)(18,26,37)(20,36,28)(22,49,27)(29,35,41)(33,44,46)} and $G/D \cong C_2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	1176	solvable	cyclic	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	1176	solvable	cyclic	C_2
2	392	solvable	dihedral	C_3
3	98	solvable	solvable	C_2^2
4	49	abelian	solvable	C_2
5	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	1176	solvable	1	(2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46)(1,22,21)(2,25,18)(3,26,17)(4,27,19)(5,28,16)(6,23,15)(7,24,20)(8,33,47)(9,30,45)(10,35,48)(11,31,43)(12,34,46)(13,29,44)(14,32,49)(36,38,39)(37,42,40)
2	784	solvable	3	(2,34,5,10)(3,42,6,18)(4,44,7,26)(8,17,29,41)(9,22,33,43)(11,13,35,31)(12,40,30,16)(14,46,32,28)(15,25,36,49)(19,21,37,39)(20,48,38,24)(23,45,47,27)(1,46,15,49)(2,39,19,35)(3,25,20,14)(4,18,21,7)(5,32,16,42)(6,11,17,28)(8,45,22,48)(9,38,26,34)(10,24,27,13)(12,31,23,41)(29,44,36,47)(30,37,40,33)

3	588	solvable	4	(2,6,4)(3,7,5)(8,23,40)(9,28,37)(10,26,42)(11,25,38)(12,27,41)(13,24,36)(14,22,39)(15,31,48)(16,29,47)(17,30,45)(18,34,44)(19,33,46)(20,35,49)(21,32,43)(1,43,30,9)(2,41,29,27)(3,28,35,38)(4,16,33,15)(5,31,32,7)(6,11,34,47)(8,10,44,49)(12,26,46,39)(13,42,48,24)(14,18,45,19)(17,36,21,23)(22,40,37,25)
4	48	solvable	49	(2,47,22)(3,13,29)(4,21,36)(5,23,43)(6,31,8)(7,39,15)(9,45,10)(11,18,17)(12,28,38)(14,40,24)(16,48,32)(19,26,25)(20,30,46)(27,34,33)(35,42,41)(37,44,49)(2,41,48,37,5,17,24,19)(3,49,14,45,6,25,32,27)(4,9,16,11,7,33,40,35)(8,34,21,20,29,10,39,38)(12,22,44,31,30,43,26,13

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	1176	solvable	cyclic	(2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46)(1,22,21)(2,25,18)(3,26,17)(4,27,19)(5,28,16)(6,23,15)(7,24,20)(8,33,47)(9,30,45)(10,35,48)(11,31,43)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	16	147	nilpotent	16	*G*	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(2,34,5,10)(3,42,6,18)(4,44,7,26)(8,17,29,41)(9,22,33,43)(11,13,35,31)(12,40,30,16)(14,46,32,28)(15,25,36,49)(19,21,37,39)(20,48,38,24)(23,45,47,27)
3	3	196	cyclic	12	1176	(2,4,6)(3,5,7)(8,40,23)(9,37,28)(10,42,26)(11,38,25)(12,41,27)(13,36,24)(14,39,22)(15,48,31)(16,47,29)(17,45,30)(18,44,34)(19,46,33)(20,49,35)(21,43,32)
7	49	1	abelian	*G*	49	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16,21,18,19,17)(22,27,23,28,25,26,24)(29,34,30,35,32,33,31)(36,41,37,42,39,40,38)(43,48,44,49,46,47,45)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)

26. Let *G* be a primitive group of degree 49 with 6 generators. We have $|G| = 3528 = 2^3 \times 3^2 \times 7^2$.

	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49) (order 2)
	a_3 = (2,5)(3,6)(4,7)(9,12)(10,13)(11,14)(16,19)(17,20)(18,21)(23,26)(24,27)(25,28)(30,33)(31,34)(32,35)(37,40)(38,41)(39,42)(44,47)(45,48)(46,49) (order 2)

Generators of G:	a_4 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45) (order 3)
	a_5 = (2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (order 3)
	a_6 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 294, generated by {(1,42,10,44,26,34,18)(2,40,13,46,22,35,17)(3,37,12,48,25,29,21)(4,36,14,45,23,33,20)(5,41,11,43,28,31,16)(6,39,8,49,24,30,19)(7,38,9,47,27,32,15)(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)} and $G/D \cong C_2^2 \times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	294	solvable	abelian	$C_2^2 \times C_3$
2	147	solvable	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	294	solvable	abelian	$C_2^2 \times C_3$
2	49	abelian	solvable	$C_2 \times C_3$
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	1764	solvable	1	(2,4,6)(3,5,7)(8,43,36,29,22,15)(9,46,41,30,25,20)(10,47,42,31,26,21)(11,48,37,32,27,16)(12,49,38,33,28,17)(13,44,39,34,23,18)(14,45,40,35,24,19)(1,30,48,40,21,25,8,2,34,47,42,18,22,9,6,33,49,39,15,23,13,5,35,46,36,16,27,12,7,32,43,37,20,26,14,4,29,44,41,19,28,11)(3,31,45,38,17,24,10)
2	1764	solvable	1	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,21,5,18,6,17)(2,16)(3,15,7,19,4,20)(8,28,12,25,13,24)(9,23)(10,22,14,26,11,27)(29,42,33,39,34,38)(30,37)(31,36,35,40,32,41)(43,49,47,46,48,45)
3	1764	solvable	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,19,8,20,36,21,29,16,43,17,22,18)(2,47,10,27,39,7,33,9,48,38,28,32)(3,26,11,6,40,14,34,37,49,31,23,46)(4,5,12,13,41,42,35,30,44,45,24,25)
				(2,6,4)(3,7,5)(8,43,36,29,22,15)(9,48,39,30,27,18)(10,49,40,31,28,19)(11,44,41,32,23,20)(12,45,42,

4	1176	solvable	1	33,24,21)(13,46,37,34,25,16)(14,47,38,35,26,17)(1,13,49,32,17,36,9,28,33,3,41,44,25,19)(2,27,47,4,20,43,11,21,29,10,42,30,24,40)(5,6,48,46,18,15,8,14,35,31,38,37,23,26)(7,34,45,39,16,22,12)
5	1176	solvable	3	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(1,29,35,28,27,6)(2,8,32,21,26,41)(3,43,31,49,24,48)(4,15,33,42,23,13)(5,36,30,14,25,20)(7,22,34)(9,11,18,19,40,37)(10,46,17,47,38,44)(12,39,16)
6	72	solvable	49	(2,6,4)(3,7,5)(8,43,36,29,22,15)(9,48,39,30,27,18)(10,49,40,31,28,19)(11,44,41,32,23,20)(12,45,42,33,24,21)(13,46,37,34,25,16)(14,47,38,35,26,17)(2,22,6,8,4,36)(3,29,7,15,5,43)(9,25,41)(10,32,42,16,26,48)(11,39,37,23,27,13)(12,46,38,30,28,20)(14,18,40,44,24)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
2	147	solvable	solvable	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
3	98	solvable	solvable	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)

4	294	solvable	dihedral	1,2,3	(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31)(1,36,15,43,8,22)(2,42,20,47,11,24)(3,37,21,48,12,25)(4,38,16,49,13,26)(5,39,17,44,14,27)(6,40,18,45,9,28)(7,41,19,46,10,23)(30,35,34,33,32,31)(1,40,32)(2,38,34)(3,41,30)(4,36,33)(5,39,29)(6,37,31)(7,42,35)(8,12,11)(9,10,13)(15,26,46)(16,24,48)(17,27,44)(18,22,47)(19,25,43)(20,23,45)(21,28,49)
5	196	solvable	solvable	1,3	(1,15)(2,19)(3,20)(4,21)(5,16)(6,17)(7,18)(8,22)(9,26)(10,27)(11,28)(12,23)(13,24)(14,25)(29,36)(30,40)(31,41)(32,42)(33,37)(34,38)(35,39)(44,47)(45,48)(46,49)(1,26,36,33,8,19,43,5,22,40,29,12,15,47)(2,28,37,35,9,21,44,7,23,42,30,14,16,49)(3,24,38,31,10,17,45)(4,27,39,34,11,20,46,6,25,41,32,13,18,48)
6	588	solvable	dihedral	1,2,3,4,5	(1,3)(2,4)(5,6)(8,10)(9,11)(12,13)(15,17)(16,18)(19,20)(22,24)(23,25)(26,27)(29,31)(30,32)(33,34)(36,38)(37,39)(40,41)(43,45)(44,46)(47,48)(1,11,44,7,10,48)(2,14,45,6,8,46)(3,13,43,4,9,49)(5,12,47)(15,32,37,21,31,41)(16,35,38,20,29,39)(17,34,36,18,30,42)(19,33,40)(22,25,23,28,24,27),(1,31,21,13,23,47)(2,33,15,10,28,48)(3,35,20,9,26,43)(4,32,18,11,25,46)(5,29,17,14,27,44)(6,30,19,8,24,49)(7,34,16,12,22,45)(36,38,42,41,37,40)
7	147	solvable	nilpotent	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,47,21,13,31,39,23)(2,43,19,14,34,38,25)(3,46,16,8,33,42,27)(4,44,15,12,35,41,24)(5,49,20,10,32,37,22)(6,45,18,9,29,40,28)(7,48,17,11,30,36,26)
8	441	solvable	nilpotent	1,2,7	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,47,42,6,45,39,2,43,40,7,48,38,4,44,36,5,49,41,3,46,37)(8,26,35,13,24,32,9,22,33,14,27,31,11,23,29,12,28,34,10,25,30)(15,19,21,20,17,18,16)
					(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,

9	294	solvable	abelian	1,3,7	31,37,49)(14,20,26,32,38,44)(1,47,21,13,31,39,23)(2,43,19,14,34,38,25)(3,46,16,8,33,42,27)(4,44,15,12,35,41,24)(5,49,20,10,32,37,22)(6,45,18,9,29,40,28)(7,48,17,11,30,36,26)
10	882	solvable	abelian	1,2,3,4,7,8,9	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,11,36,32,43,25)(2,9,37,30,44,23)(3,12,38,33,45,26)(4,8,39,29,46,22)(5,10,40,31,47,24)(6,14,41,35,48,28)(7,13,42,34,49,27)(15,18)(17,19)(20,21)
11	588	solvable	cyclic	1,3,7,9	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,36,37,2)(3,29,42,44)(4,15,40,9)(5,8,39,16)(6,22,41,23)(7,43,38,30)(10,32,21,47)(11,18,19,12)(13,25,20,26)(14,46,17,33)(24,34,28,48)(31,35,49,45)
12	588	solvable	cyclic	1,3,5,7,9	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,47,34,22,40,20)(2,44,30,23,37,16)(3,49,32,24,42,18)(4,45,35,25,38,21)(5,48,29,26,41,15)(6,43,33,27,36,19)(7,46,31,28,39,17)(8,12,13)(10,14,11)
13	1764	solvable	cyclic	1,2,3,4,7,8,9,10,11	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,36,39,25,23,16,21,35,31,10,13,6)(2,15,42,32,24,9,20,7,29,38,11,27)(3,8,41,4,22,37,18,28,30,17,14,34)(5,43,40,46,26,44,19,49,33,45,12,48)
14	1764	solvable	cyclic	1,2,3,4,5,6,7,8,9,10,12	(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)(1,39,27,19,30,10,7,36,25,20,33,9,3,42,22,18,34,12,2,38,28,15,32,13,5,37,24,21,29,11,6,40,23,17,35,8,4,41,26,16,31,14)(43,46,48,47,44,45,49)
15	588	solvable	cyclic	1,3,7,9	(2,29)(3,36)(4,43)(5,8)(6,15)(7,22)(9,33)(10,40)(11,47)(13,19)(14,26)(16,34)(17,41)(18,48)(21,27)(23,35)(24,42)(25,49)(31,37)(32,44)(39,45)(1,15,21,35,31,24,23,44,47,12,13,41,39,4)(2,43,19,14,34,38,25)(3,22,16,49,33,10,27,37,46,5,8,20,42,32)(6,36,18,7,

					29,17,28,30,45,26,9,48,40,11)
16	1764	solvable	cyclic	1,2,3,4,7,8,9,10	(2,29)(3,36)(4,43)(5,8)(6,15)(7,22)(9,33)(10,40)(11,47)(13,19)(14,26)(16,34)(17,41)(18,48)(21,27)(23,35)(24,42)(25,49)(31,37)(32,44)(39,45)(1,29,31,24,23,2)(3,22,30)(4,8,34,17,28,37)(5,43,33,45,26,44)(6,15,35,38,25,9)(7,36,32,10,27,16)(11,13,20,21,42,39)(12,48,19,49,40,46)(14,41,18)
17	1176	solvable	cyclic	--	(2,6,4)(3,7,5)(8,43,36,29,22,15)(9,48,39,30,27,18)(10,49,40,31,28,19)(11,44,41,32,23,20)(12,45,42,33,24,21)(13,46,37,34,25,16)(14,47,38,35,26,17)(1,34,28,11,38,15,30,49,12,3,20,23,46,40)(2,48,26,4,41,22,32,42,8,31,21,9,45,19)(5,6,27,25,39,36,29,35,14,10,17,16,44,47)(7,13,24,18,37,43,33)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	147	nilpotent	24	1176	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
3	9	49	abelian	72	441	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
7	49	1	abelian	*G*	49	(1,22,36,29,8,15,43)(2,23,37,30,9,16,44)(3,24,38,31,10,17,45)(4,25,39,32,11,18,46)(5,26,40,33,12,19,47)(6,27,41,34,13,20,48)(7,28,42,35,14,21,49)(1,32,48,40,16,24,14)(2,31,49,36,18,27,12)(3,35,43,39,20,26,9)(4,34,47,37,17,28,8)(5,30,45,42,15,25,13)(6,33,44,38,21,22,11)(7,29,46,41,19,23,10)

27. Let G be a primitive group of degree 49 with 5 generators. We have $|G| = 3528 = 2^3\times3^2\times7^2$.

Generators of *G*:	a_1 = (2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24)(7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39,23)(17,33,49)(20,26,27)(21,47,31)(28,34,35)(30,42,37)(38,44,45) (order 3)
	a_2 = (2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46) (order 4)
	a_3 = (2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46) (order 4)

	a_4 = (2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (order 3)
	a_5 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. *G* is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup *D* = [*G*, *G*] is a solvable group of order 392, generated by {(2,24,5,48)(3,32,6,14)(4,40,7,16)(8,39,29,21)(9,35,33,11)(10,20,34,38)(12,26,30,44)(13,43,31,22)(15,47,36,23)(17,37,41,19)(18,28,42,46)(25,45,49,27)(1,39,37,5)(2,13,36,20)(3,23,42,22)(4,47,40,32)(6,31,41,49)(7,21,38,10)(8,26,16,25)(9,30,15,43)(11,17,19,14)(12,48,18,34)(24,46,28,33)(29,35,44,45)} and $G/D \cong C_3^2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	abelian	C_3^2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	abelian	C_3^2
2	98	solvable	solvable	C_2^2
3	49	abelian	solvable	C_2
4	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	1176	solvable	1	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(1,22,20,3,24,15,5,26,17,4,25,19,7,28,18,2,23,21,6,27,16)(8,32,43,10,35,45,12,30,47,11,34,46,14,29,49,9,31,44,13,33,48)(36,37,39,38,41,42,40)
2	1176	solvable	1	(2,39,5,21)(3,47,6,23)(4,13,7,31)(8,48,29,24)(9,26,33,44)(10,17,34,41)(11,30,35,12)(14,36,32,15)(16,43,40,22)(18,25,42,49)(19,38,37,20)(27,46,45,28)(1,8,4)(2,36,37)(3,15,47)(5,43,17)(6,29,28)(7,22,34)(9,18,40)(10,25,48)(11,32,14)(12,39,16)(13,46,24)(19,26,20)(21,33,35)(23,27,41)(30,49,38)(31,42,44)
3	1176	solvable	1	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,33,39,12,23,5,21,40,31,26,13,19)(2,22,42,18,24,30,20,14,29,3,11,41)(4,44,37,49,28,45,17,48,34,43,8,46)(6,38,36,27,25,15,16,32,35,9,10,7)
4	1176	solvable	1	(2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46)(1,22,26,9,16,15,25,39,37,17,45,46,41,34,31,49,7,6,33,12,14)(2,21,5,11,40,8,24,43,23,20,30,18,42,4,38,47,10,48,29,27,35)(3,44,19,13,32,36,28)

5	882	solvable	4	(1,43,20,16,44,6)(2,22,3,15,9,21)(4,29,39,19,37,33)(5,8,35,18,23,38)(7,36,26,17,30,11)(10,42,12,28,31,25)(13,14,49,27,24,45)(32,46,41,40,47,34)(1,43,26,23,2,36,39,25,30,31,38,11,13,34,17,21,14,48,47,19,7)(3,22,9,27,37,18,42,32,45,33,10,6,8,20,28,16,49,40,46,5,29)(4,15,35,24,44,12,41)
6	72	solvable	49	(8,39,24)(9,38,27)(10,42,26)(11,41,28)(12,37,25)(13,40,22)(14,36,23)(15,47,32)(16,43,31)(17,46,35)(18,44,34)(19,49,30)(20,45,33)(21,48,29)(2,32,13)(3,40,21)(4,48,23)(5,14,31)(6,16,39)(7,24,47)(8,36,22)(9,18,20)(10,12,49)(11,27,37)(15,43,29)(17,26,28)(19,35,45)(25,34,30)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
2	147	solvable	solvable	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
3	98	solvable	solvable	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
4	294	solvable	solvable	1,2,3	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)

5	392	solvable	abelian	1,3	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,10,31)(2,6,11,12)(3,21,8,49)(4,44,9,18)(5,38,13,36)(7,33,14,27)(15,37,45,39)(16,28,46,35)(17,47,43,20)(19,34,48,26)(23,29,32,24)(25,41,30,40)
6	1176	solvable	cyclic	1,2,3,4,5	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,5,33,3,38,2,44,4,11,7,21)(8,10,19,16,24,25,30,35,39,36,49,47)(9,41,18,48,28,13,29,20,40,27,45,34)(12,46,17,14,23,15,32,26,42,31,43,37)
7	1176	solvable	cyclic	1,3,5	(2,24,5,48)(3,32,6,14)(4,40,7,16)(8,39,29,21)(9,35,33,11)(10,20,34,38)(12,26,30,44)(13,43,31,22)(15,47,36,23)(17,37,41,19)(18,28,42,46)(25,45,49,27)(1,8,11,30,2,3,46,32,34,38,45,49,20,41,40,28,21,15,12,26,23)(4,33,25,31,43,10,48,37,6,42,18,35,19,24,47,22,13,36,9,7,16)(5,27,39,29,14,17,44)
8	1176	solvable	cyclic	1,3,5	(2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46)(1,22,20,3,24,15,5,26,17,4,25,19,7,28,18,2,23,21,6,27,16)(8,32,43,10,35,45,12,30,47,11,34,46,14,29,49,9,31,44,13,33,48)(36,37,39,38,41,42,40)
9	1176	solvable	cyclic	1,3,5	(2,39,5,21)(3,47,6,23)(4,13,7,31)(8,48,29,24)(9,26,33,44)(10,17,34,41)(11,30,35,12)(14,36,32,15)(16,43,40,22)(18,25,42,49)(19,38,37,20)(27,46,45,28)(1,8,4)(2,36,37)(3,15,47)(5,43,17)(6,29,28)(7,22,34)(9,18,40)(10,25,48)(11,32,14)(12,39,16)(13,46,24)(19,26,20)(21,33,35)(23,27,41)(30,49,38)(31,42,44)

Sylow subgroups						
p	Ordre	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	nilpotent	72	392	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)
3	9	196	abelian	18	*G*	(2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24)(7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39,23)(17,33,49)(20,26,27)(21,47,31)(28,34,35)(30,42,37)(38,44,45)(2,29,24)(3,36,32)(4,43,40)(5,8,48)(6,15,14)(7,22,16)(9,41,25)(10,27,30)(11,20,42)(12,34,45)(17,49,33)(18,35,38)

						(19,28,44)(26,37,46)
7	49	1	abelian	*G*	49	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16,21,18,19,17)(22,27,23,28,25,26,24)(29,34,30,35,32,33,31)(36,41,37,42,39,40,38)(43,48,44,49,46,47,45)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)

28. Let G be a primitive group of degree 49 with 3 generators. We have $|G| = 4704 = 2^5\times3\times7^2$.

Generators of G:	a_1 = (2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44) (order 2)
	a_2 = (2,40,21,35,30,42,33,15,3,48,23,37,38,44,41,22,4,14,31,45,46,10,49,29,5,16,39,11,12,18,9,36,6,24,47,19,20,26,17,43,7,32,13,27,28,34,25,8) (order 48)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 392, generated by {(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)} and $G/D \cong C_2^2\times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	abelian	$C_2^2\times C_3$
2	196	solvable	nilpotent	C_2
3	98	solvable	nilpotent	C_2
4	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	abelian	$C_2^2\times C_3$
2	49	abelian	nilpotent	C_8
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	2352	solvable	1	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16,21,18,19,17)(22,27,23,28,25,26,24)(29,34,30,35,32,33,31)(36,41,37,42,39,40,38)(43,48,44,49,46,47,45)(2,40,21,35,30,42,33,15,3,48,23,37,38,44,41,22,4,14,31,45,46,10,49,29,5,16,39,11,12,18,9,36,6,24,47,19,20,26,17,43,7,32,13,27,28,34,25,8)
2	2352	solvable	1	(2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44)(1,46,28,12,20,30)(2,48,25,8,19,35)(3,44,24,11,17,33)

				(4,47,26,9,16,32)(5,43,23,14,18,34)(6,45,22,10,21,31)(7,49,27,13,15,29)(36,41,42)(37,40,39)
3	2352	solvable	1	(2,34,5,10)(3,42,6,18)(4,44,7,26)(8,17,29,41)(9,22,33,43)(11,13,35,31)(12,40,30,16)(14,46,32,28)(15,25,36,49)(19,21,37,39)(20,48,38,24)(23,45,47,27)(1,23,40,43,46,8,41,25,28,18,37,14,12,35,36,19,20,5,39,34,30,48,42,2)(3,21,49,27,45,33,26,9,24,6,13,15,10,44,16,32,17,22,29,7,31,11,4,47)
4	1568	solvable	1	(2,24,31,35,20,10,33,43,5,48,13,11,38,34,9,22)(3,32,39,37,28,18,41,8,6,14,21,19,46,42,17,29)(4,40,47,45,30,26,49,15,7,16,23,27,12,44,25,36)(1,6,2,7,4,5,3)(8,33,9,29,11,30,10,32,13,31,14,34,12,35)(15,38,16,41,18,42,17,40,20,36,21,37,19,39)(22,44,23,46,25,45,24,48,27,49,28,47,26,43)
5	96	nilpotent	49	(2,24,31,35,20,10,33,43,5,48,13,11,38,34,9,22)(3,32,39,37,28,18,41,8,6,14,21,19,46,42,17,29)(4,40,47,45,30,26,49,15,7,16,23,27,12,44,25,36)(2,4,6)(3,5,7)(8,48,36,32,22,16)(9,47,41,31,25,21)(10,45,42,35,26,19)(11,44,37,34,27,18)(12,46,38,30,28,20)(13,49,39,33,23,

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)
2	147	solvable	nilpotent	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
3	98	solvable	nilpotent	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)

4	294	solvable	nilpotent	1,2,3	(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
5	196	solvable	nilpotent	1,3	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
6	588	solvable	nilpotent	1,2,3,4,5	(1,46,20,12,30,38,28)(2,45,21,8,32,41,26)(3,49,15,11,34,40,23)(4,48,19,9,31,42,22)(5,44,17,14,29,39,27)(6,47,16,10,35,36,25)(7,43,18,13,33,37,24)(2,12,3,20,4,28,5,30,6,38,7,46)(8,11,15,19,22,27,29,35,36,37,43,45)(9,23,17,31,25,39,33,47,41,13,49,21)(10,40,18,48,26,14,34,16,42,24,44,32)
7	392	solvable	abelian	1,3,5	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
8	784	solvable	cyclic	1,3,5,7	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,37,42,6)(2,19,41,12)(3,11,39,17)(4,27,38,23)(5,45,40,32)(7,35,36,43)(8,33,21,47)(9,48,20,30)(10,15,18,14)(13,28,16,22)(24,49,25,29)(31,34,46,44)
9	784	solvable	cyclic	1,3,5,7	(2,24,31,35,20,10,33,43,5,48,13,11,38,34,9,22)(3,32,39,37,28,18,41,8,6,14,21,19,46,42,17,29)(4,40,47,45,30,26,49,15,7,16,23,27,12,44,25,36)(1,32,48,40,16,24,14)(2,31,49,36,18,27,12)(3,35,43,39,20,26,9)(4,34,47,37,17,28,8)(5,30,45,42,15,25,13)(6,33,44,38,21,22,11)(7,29,46,41,19,23,10)
10	1176	solvable	abelian	1,2,3,4,5,6,7	(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)(2,47,46,33,7,39,38,25,6,31,30,17,5,23,28,9,4,21,20,49,3,13,12,41)(8,24,45,42,43,16,37,34,36,14,35,26,29,48,27,18,22,40,19,10,15,32,11,44)

11	2352	solvable	cyclic	1,2,3,4,5,6,7,8,10	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,33,36,13,22,7,15,37,29,24,8,18)(2,39,38,26,25,20,19,35,34,9,14,3)(4,28,40,16,27,31,21,11,30,5,10,41)(6,48,42,49,23,44,17,45,32,46,12,47)
12	2352	solvable	cyclic	1,2,3,4,5,6,7,9,10	(2,40,21,35,30,42,33,15,3,48,23,37,38,44,41,22,4,14,31,45,46,10,49,29,5,16,39,11,12,18,9,36,6,24,47,19,20,26,17,43,7,32,13,27,28,34,25,8)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
13	784	solvable	cyclic	1,3,5,7	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(1,25,22,39,36,32,29,11,8,18,15,46,43,4)(2,9,23,16,37,44,30)(3,33,24,12,38,19,31,47,10,5,17,26,45,40)(6,49,27,7,41,28,34,42,13,35,20,14,48,21)
14	1568	solvable	cyclic	1,3,5,7,8,9,13	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(1,34,7,28,26,39,21,24,43,31,46,18,16,14,25,20)(2,8,30,10,45,49,27,3,47,36,33,41,6,4,17,48)(5,37,11,15,32,19,40,38,44,12,42,22,35,23,9,13)
15	2352	solvable	cyclic	1,2,3,4,5,6,7,10	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,48,27,12,19,29)(2,47,23,8,16,34)(3,45,25,11,21,35)(4,44,28,9,17,30)(5,46,22,14,20,31)(6,49,26,10,15,32)(7,43,24,13,18,33)(36,40,41)(38,42,39)

Sylow subgroups						
p	Ordre	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	32	49	nilpotent	96	1568	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(2,24,31,35,20,10,33,43,5,48,13,11,38,34,9,22)(3,32,39,37,28,18,41,8,6,14,21,19,46,42,17,29)(4,40,47,45,30,26,49,15,7,16,23,27,12,44,25,36)
3	3	49	cyclic	96	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
						(1,33,49,41,17,25,9)(2,29,47,42,20,24,11)(3,32,44,36,19,28,13)(4,30,43,40,21,27,10)(5,35,48,38,18,23,8)(6,31,46,37,15,26,14)

7	49	1	abelian	G	49	(7,34,45,39,16,22,12)(1,43,15,8,29,36,22) (2,44,16,9,30,37,23)(3,45,17,10,31,38,24) (4,46,18,11,32,39,25)(5,47,19,12,33,40,26) (6,48,20,13,34,41,27)(7,49,21,14,35,42,28)

29. Let G be a primitive group of degree 49 with 6 generators. We have $|G| = 7056 = 2^4\times3^2\times7^2$.

Generators of G:	a_1 = (2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25) (order 4)
	a_2 = (2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24)(7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39,23)(17,33,49)(20,26,27)(21,47,31)(28,34,35)(30,42,37)(38,44,45) (order 3)
	a_3 = (2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46) (order 4)
	a_4 = (2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46) (order 4)
	a_5 = (2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (order 3)
	a_6 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup D = [G, G] is a solvable group of order 1176, generated by {(2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46)(1,18,38)(2,40,21)(3,29,11)(4,45,22)(5,28,44)(7,9,33)(8,46,10)(12,42,23)(13,27,41)(14,16,19)(15,25,24)(17,43,32)(20,34,48)(26,35,30)(31,36,39)(37,47,49)} and $G/D \cong C_2\times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	1176	solvable	cyclic	$C_2\times C_3$

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	1176	solvable	cyclic	$C_2\times C_3$
2	392	solvable	solvable	C_3
3	98	solvable	solvable	C_2^2
4	49	abelian	solvable	C_2
5	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	3528	solvable	1	(2,16,3,24,4,32,5,40,6,48,7,14)(8,31,15,39,22,47,29,13,36,21,43,23)(9,27,17,35,25,37,33,45,41,11,49,19)(10,12,18,20,26,28,34,30,42,38,44,46)(1,33,39,5,35,37,7,34,36,6,31,40,3,32,42,4,30,41,2,29,38)(8,11,13,12,9,10,14)(15,44,23,19,43,22,21,47,

				26,20,49,28,17,48,27,18,45,24,16,46,25)
2	2352	solvable	1	(2,47,22)(3,13,29)(4,21,36)(5,23,43)(6,31,8)(7,39,15)(9,45,10)(11,18,17)(12,28,38)(14,40,24)(16,48,32)(19,26,25)(20,30,46)(27,34,33)(35,42,41)(37,44,49)(1,2,42,25,31,32,28,37)(3,47,45,7,29,13,8,35)(4,11,5,20,30,44,34,19)(6,24,15,40,33,36,17,27)(9,18,43,22,46,16,10,38)(12,23,26,14,48,39,41,49)
3	2352	solvable	3	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,38,18,13,44,29,42,4,10,16,34,49)(2,31,21,6,45,15,41,46,8,37,32,14)(3,17,20,48,43,36,39,11,9,30,35,7)(5,24,19,27,47,22,40,25,12,23,33,28)
4	1764	solvable	4	(2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24)(7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39,23)(17,33,49)(20,26,27)(21,47,31)(28,34,35)(30,42,37)(38,44,45)(1,19,47,3,31,44,23,32,39,28,21,36)(2,42,46,15,35,5,22,45,40,30,17,25)(4,48,49,34,29,27,26,41,38,20,16,6)(7,9,43,11,33,14,24,8,37,12,18,10)
5	144	solvable	49	(2,40,15)(3,48,22)(4,14,29)(5,16,36)(6,24,43)(7,32,8)(9,41,25)(10,46,11)(12,19,18)(13,23,39)(17,49,33)(20,27,26)(21,31,47)(28,35,34)(30,37,42)(38,45,44)(2,33,32,19,7,25,24,11,6,17,16,45,5,9,14,37,4,49,48,35,3,41,40,27)(8,26,47,38,43,18,39,30,36,10,31,28,29,44,23,20,22,

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
2	147	solvable	solvable	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
					(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)

3	98	solvable	solvable	1	(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
4	294	solvable	solvable	1,2,3	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
5	392	solvable	solvable	1,3	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,10,31)(2,6,11,12)(3,21,8,49)(4,44,9,18)(5,38,13,36)(7,33,14,27)(15,37,45,39)(16,28,46,35)(17,47,43,20)(19,34,48,26)(23,29,32,24)(25,41,30,40)
6	1176	solvable	dihedral	1,2,3,4,5	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,5,33,3,38,2,44,4,11,7,21)(8,10,19,16,24,25,30,35,39,36,49,47)(9,41,18,48,28,13,29,20,40,27,45,34)(12,46,17,14,23,15,32,26,42,31,43,37)
7	1176	solvable	cyclic	1,3,5	(2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46)(1,22,21)(2,25,18)(3,26,17)(4,27,19)(5,28,16)(6,23,15)(7,24,20)(8,33,47)(9,30,45)(10,35,48)(11,31,43)(12,34,46)(13,29,44)(14,32,49)(36,38,39)(37,42,40)
8	3528	solvable	cyclic	1,2,3,4,5,6,7	(2,4,6)(3,5,7)(8,40,23)(9,37,28)(10,42,26)(11,38,25)(12,41,27)(13,36,24)(14,39,22)(15,48,31)(16,47,29)(17,45,30)(18,44,34)(19,46,33)(20,49,35)(21,43,32)(1,22,25,14,21,15,24,38,42,16,44,45,40,33,30,48,6,5,32,11,13)(2,49,18,12,31,36,27)(3,37,46,9,47,29,26,34,7,20,4,10,39,8,23,43,28,19,35,17,41)

9	2352	solvable	cyclic	1,3,5,7	(2,32,13)(3,40,21)(4,48,23)(5,14,31)(6,16,39)(7,24,47)(8,36,22)(9,18,20)(10,12,49)(11,27,37)(15,43,29)(17,26,28)(19,35,45)(25,34,30)(33,42,38)(41,44,46)(1,22,2,34,49,21,48,30)(3,7,12,36,46,43,40,14)(4,16,18,5,45,27,24,47)(6,39,31,9,44,10,32,41)(8,13,19,23,42,37,26,20)(11,28,29,17,38,15,35,25)

Sylow subgroups						
p	Ordre	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	16	147	nilpotent	48	2352	(2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46)(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)
3	9	196	abelian	36	3528	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(2,40,15)(3,48,22)(4,14,29)(5,16,36)(6,24,43)(7,32,8)(9,41,25)(10,46,11)(12,19,18)(13,23,39)(17,49,33)(20,27,26)(21,31,47)(28,35,34)(30,37,42)(38,45,44)
7	49	1	abelian	*G*	49	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)(1,34,44,42,18,26,10)(2,35,46,40,17,22,13)(3,29,48,37,21,25,12)(4,33,45,36,20,23,14)(5,31,43,41,16,28,11)(6,30,49,39,19,24,8)(7,32,47,38,15,27,9)

30. Let *G* be the primitive group 7^2: *GL*(2,7) ◊ 1. We have $|G| = 49 = 7^2$. *G* is a cyclic group generated by (1,8,22,15,36,43,29), (2,9,23,16,37,44,30), (3,10,24,17,38,45,31), (4,11,25,18,39,46,32), (5,12,26,19,40,47,33), (6,13,27,20,41,48,34), (7,14,28,21,42,49,35).

Non-trivial orbits of *G*: {1,8,22,15,36,43,29}, {2,9,23,16,37,44,30}, {3,10,24,17,38,45,31}, {4,11,25,18,39,46,32}, {5,12,26,19,40,47,33}, {6,13,27,20,41,48,34}, {7,14,28,21,42,49,35}.

31. Let *G* be the primitive group 7^2: *GL*(2,7) ◊ 2. We have $|G| = 98 = 2\times7^2$.

Generators of *G*:	a_1 = (8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49) (order 2)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

G is a solvable group. Non-trivial orbits of *G*: {1,8,29,22,43,15,36}, {2,9,30,23,44,16,37}, {3,10,31,24,45,17,38}, {4,11,32,25,46,18,39}, {5,12,33,26,47,19,40}, {6,13,34,27,48,20,41}, {7,14,35,28,49,21,42}. The center of *G* is trivial.

The derived subgroup *D* = [*G*, *G*] is a cyclic group of order 7, generated by {(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)} and $G/D \cong C_2$.

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	-
2	2	cyclic	7	1	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
3	7	cyclic	1	1	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
4	14	dihedral	1	2,3	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	7	cyclic	cyclic	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)

Sylow subgroups						
p	Ordre	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	2	7	cyclic	2	14	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
7	7	1	cyclic	14	7	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)

32. Let *G* be the primitive group 7^2: *GL*(2,7) ◊ 3. We have $|G| = 147 = 3\times7^2$.

Generators of *G*:	a_1 = (8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49) (order 3)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

G is a solvable group. Non-trivial orbits of *G*: {1,8,22,36,15,43,29}, {2,9,23,37,16,44,30}, {3,10,24,38,17,45,31}, {4,11,25,39,18,46,32}, {5,12,26,40,19,47,33}, {6,13,27,41,20,48,34}, {7,14,28,42,21,49,35}. The center of *G* is trivial.

The derived subgroup *D* = [*G*, *G*] is a cyclic group of order 7, generated by {(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)} and $G/D \cong C_3$.

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	-
2	3	cyclic	7	1	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49)
3	7	cyclic	1	1	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
4	21	solvable	1	2,3	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	7	cyclic	cyclic	(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)

Sylow subgroups						
p	Ordre	Conjug. classes	Nature	Normalizer	Normal closure	Generators
3	3	7	cyclic	3	21	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49)
7	7	1	cyclic	21	7	(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)(7,35,49,42,21,28,14)

33. Let G be the primitive group 7^2: $GL(2,7)$ ◊ 4. We have $|G| = 294 = 2\times3\times7^2$.

Generators of G:	a_1 = (8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49) (order 3)
	a_2 = (8,15,22,29,36,43)(9,16,23,30,37,44)(10,17,24,31,38,45)(11,18,25,32,39,46)(12,19,26,33,40,47)(13,20,27,34,41,48)(14,21,28,35,42,49) (order 6)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. Non-trivial orbits of G: {1,8,22,15,36,29,43}, {2,9,23,16,37,30,44}, {3,10,24,17,38,31,45}, {4,11,25,18,39,32,46}, {5,12,26,19,40,33,47}, {6,13,27,20,41,34,48}, {7,14,28,21,42,35,49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a cyclic group of order 7, generated by {(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)} and $G/D \cong C_2 \times C_3$.

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	7	cyclic	cyclic	$C_2 \times C_3$
2	1	trivial	solvable	C_7

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	-
2	2	cyclic	7	1	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
3	3	cyclic	7	1	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49)
4	6	cyclic	7	2,3	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49)(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
5	7	cyclic	1	1	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
6	14	dihedral	1	2,5	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
7	21	solvable	1	--	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
8	42	solvable	1	--	(8,43,36,29,22,15)(9,44,37,30,23,16)(10,45,38,31,24,17)(11,46,39,32,25,18)(12,47,40,33,26,19)(13,48,41,34,27,20)(14,49,42,35,28,21)(1,22,36,29,8,15,43)(2,23,37,30,9,16,44)(3,24,38,31,10,17,45)(4,25,39,32,11,18,46)(5,26,40,33,12,19,47)(6,27,41,34,13,20,48)(7,28,42,35,14,21,49)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	21	solvable	cyclic	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49)(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)(7,35,49,42,21,28,14)
2	14	dihedral	cyclic	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)

Sylow subgroups						
p	Ordre	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	2	7	cyclic	6	14	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
3	3	7	cyclic	6	21	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49)
7	7	1	cyclic	42	7	(1,22,36,29,8,15,43)(2,23,37,30,9,16,44)(3,24,38,31,10,17,45)(4,25,39,32,11,18,46)(5,26,40,33,12,19,47)(6,27,41,34,13,20,48)(7,28,42,35,14,21,49)

34. Let *G* be the primitive group *L*2, 7^2 # 49.1. We have $|G| = 56448 = 2^7 \times 3^2 \times 7^2$.

Generators of *G*:	a_1 = (1,2,4)(3,7,5)(8,9,11)(10,14,12)(15,16,18)(17,21,19)(22,23,25)(24,28,26)(29,30,32)(31,35,33)(36,37,39)(38,42,40)(43,44,46)(45,49,47) (order 3)
	a_2 = (1,8,22)(2,9,23)(3,10,24)(4,11,25)(5,12,26)(6,13,27)(7,14,28)(15,43,29)(16,44,30)(17,45,31)(18,46,32)(19,47,33)(20,48,34)(21,49,35) (order 3)
	a_3 = (1,4,7)(2,6,5)(8,11,14)(9,13,12)(15,18,21)(16,20,19)(22,25,28)(23,27,26)(29,32,35)(30,34,33)(36,39,42)(37,41,40)(43,46,49)(44,48,47) (order 3)
	a_4 = (1,22,43)(2,23,44)(3,24,45)(4,25,46)(5,26,47)(6,27,48)(7,28,49)(8,36,29)(9,37,30)(10,38,31)(11,39,32)(12,40,33)(13,41,34)(14,42,35) (order 3)
	a_5 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)

This generating set is not minimal. There is a smaller generating set with 2 elements. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup *D* = [*G*, *G*] is a perfect group of order 28224, generated by {(1,24,20,44,4,26,21,43,3,27,16,46,5,28,15,45,6,23,18,47,7,22,17,48,2,25,19,49)(8,10,13,9,11,12,14)(29,38,34,37,32,40,35,36,31,41,30,39,33,42)(1,35,18,24,43,42,11,3,29,21,25,45,36,14,4,31,15,

28,46,38,8,7,32,17,22,49,39,10)(2,34,16,27,44,41,9,6,30,20,23,48,37,13)(5,33,19,26,47,40,12)}
and $G/D \cong C_2$.

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	28224	perfect	cyclic	(1,5,2,7)(4,6)(8,47,23,14,43,26,9,49,22,12,44,28)(10,45,24)(11,48,25,13,46,27)(15,40,30,21,36,33,16,42,29,19,37,35)(17,38,31)(18,41,32,20,39,34),(1,20,2,15,6,16)(3,21,5,17,7,19)(4,18)(8,13,9)(10,14,12)(22,41,44,29,27,37,43,34,23,36,48,30)(24,42,47,31,28,

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	128	441	nilpotent	128	*G*	(8,15)(9,16)(10,17)(11,18)(12,19)(13,20)(14,21)(36,43)(37,44)(38,45)(39,46)(40,47)(41,48)(42,49)(2,36)(3,43)(4,15)(5,8)(6,29)(7,22)(9,40)(10,47)(11,19)(13,33)(14,26)(16,39)(17,46)(20,32)(21,25)(23,42)(24,49)(27,35)(30,41)(31,48)(38,44)(2,6)(3,7)(9,13)(10,14)(16,20)(17,21)(23,27)(24,28)(30,34)(31,35)(37,41)(38,42)(44,48)(45,49)
3	9	784	abelian	72	28224	(8,36,22)(9,37,23)(10,38,24)(11,39,25)(12,40,26)(13,41,27)(14,42,28)(15,43,29)(16,44,30)(17,45,31)(18,46,32)(19,47,33)(20,48,34)(21,49,35)(2,4,7)(3,5,6)(9,11,14)(10,12,13)(16,18,21)(17,19,20)(23,25,28)(24,26,27)(30,32,35)(31,33,34)(37,39,42)(38,40,41)(44,46,49)(45,47,48)
7	49	64	abelian	882	28224	(1,6,3,2,5,7,4)(8,13,10,9,12,14,11)(15,20,17,16,19,21,18)(22,27,24,23,26,28,25)(29,34,31,30,33,35,32)(36,41,38,37,40,42,39)(43,48,45,44,47,49,46)(1,42,16,48,25,33,10)(2,41,18,47,24,29,14)(3,36,21,44,27,32,12)(4,40,17,43,28,30,13)(5,38,15,49,23,34,11)(6,39,19,45,22,35,9)(7,37,20,46,26,31,8)

35. Let G be the primitive group $A7$ on 1-sets[2.1]. We have $|G| = 12700800 = 2^7\times3^4\times5^2\times7^2$.

Generators of *G*:	a_1 = (1,2,3,4,5,6,7)(8,9,10,11,12,13,14)(15,16,17,18,19,20,21)(22,23,24,25,26,27,28)(29,30,31,32,33,34,35)(36,37,38,39,40,41,42)(43,44,45,46,47,48,49) (order 7)
	a_2 = (1,8,15,22,29,36,43)(2,9,16,23,30,37,44)(3,10,17,24,31,38,45)(4,11,18,25,32,39,46)(5,12,19,26,33,40,47)(6,13,20,27,34,41,48)(7,14,21,28,35,42,49) (order 7)
	a_3 = (5,6,7)(12,13,14)(19,20,21)(26,27,28)(33,34,35)(40,41,42)(47,48,49) (order 3)
	a_4 = (29,36,43)(30,37,44)(31,38,45)(32,39,46)(33,40,47)(34,41,48)(35,42,49) (order 3)
	a_5 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)

This generating set is not minimal. There is a smaller generating set with 2 elements. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a perfect group of order 6350400, generated by {(1,11,47,21,2,8,46,19,7,9,43,18,5,14,44,15,4,12,49,16)(3,10,45,17)(6,13,48,20)(22,25,26,28,23)(29,39,33,42,30,36,32,40,35,37)(31,38)(34,41)(1,28,43,35,36,14,15,7,22,49,29,42,8,21)(2,23,44,30,37,9,16)(3,26,46,34,38,12,18,6,24,47,32,41,10,19,4,27,45,33,39,13,17,5,25,48,31,40,11,20)} and $G/D \cong C_2$.

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	6350400	perfect	cyclic	C_2

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	6350400	perfect	cyclic	(1,46,34,23,12,42,17)(2,47,35,24,8,39,20)(3,43,32,27,9,40,21)(4,48,30,26,14,38,15)(5,49,31,22,11,41,16)(6,44,33,28,10,36,18)(7,45,29,25,13,37,19)(1,49,12,22,7,47,8,28,5,43,14,26)(2,45,11,23,3,46,9,24,4,44,10,25)(6,48,13,27)(15,42,19,36,21,40)

Sylow subgroups						
p	Ordre	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	128	99225	nilpotent	128	G	(15,22)(16,23)(17,24)(18,25)(19,26)(20,27)(21,28)(36,43)(37,44)(38,45)(39,46)(40,47)(41,48)(42,49), (8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(22,36)(23,37)(24,38)(25,39)(26,40)(27,41)(28,42)(2,15)(3,8)(4,43)(5,22)(6,29)(7,36)(9,17)(11,45)(12,24)(13,31)(14,38)(18,44)(19,23)(20,30)(21,37)(25,47)(27,33)(28,40)(32,48)(35,41)(39,49)
3	81	4900	abelian	2592	6350400	(3,7,6)(10,14,13)(17,21,20)(24,28,27)(31,35,34)(38,42,41)(45,49,48)(2,4,5)(9,11,12)(16,18,19)(23,25,26)(30,32,33)(37,39,40)(44,46,47), (22,43,36)(23,44,37)(24,45,38)(25,46,39)(26,47,40)(27,48,41)(28,49,42)(8,29,15)(9,30,16)(10,31,17)(11,32,18)(12,33,19)(13,34,20)(14,35,21)
5	25	15876	abelian	800	6350400	(15,22,29,36,43)(16,23,30,37,44)(17,24,31,38,45)(18,25,32,39,46)(19,26,33,40,47)(20,27,34,41,48)(21,28,35,42,49)(3,5,6,7,4)(10,12,13,14,11)(17,19,20,21,18)(24,26,27,28,25)(31,33,34,35,32)(38,40,41,42,39)(45,47,48,49,46)
7	49	14400	abelian	882	6350400	(1,5,7,4,6,2,3)(8,12,14,11,13,9,10)(15,19,21,18,20,16,17)(22,26,28,25,27,23,24)(29,33,35,32,34,30,31)(36,40,42,39,41,37,38)(43,47,49,46,48,44,45)(1,23,39,33,45,20,14)(2,25,40,31,48,21,8)(3,27,42,29,44,18,

						12)(4,26,38,34,49,15,9)(5,24,41,35, 43,16,11)(6,28,36,30,46,19,10) (7,22,37,32,47,17,13)

36. Let G be the primitive group $A7$ on 1-sets[2.2]. We have $|G| = 25401600 = 2^8\times 3^4\times 5^2\times 7^2$.

Generators of G:	a_1 = (1,2,3,4,5,6,7)(8,9,10,11,12,13,14)(15,16,17,18,19,20,21) (22,23,24,25,26,27,28)(29,30,31,32,33,34,35)(36,37,38,39,40, 41,42)(43,44,45,46,47,48,49) (order 7)
	a_2 = (1,8,15,22,29,36,43)(2,9,16,23,30,37,44)(3,10,17,24,31, 38,45)(4,11,18,25,32,39,46)(5,12,19,26,33,40,47)(6,13,20, 27,34,41,48)(7,14,21,28,35,42,49) (order 7)
	a_3 = (5,6,7)(12,13,14)(19,20,21)(26,27,28)(33,34,35) (40,41,42)(47,48,49) (order 3)
	a_4 = (29,36,43)(30,37,44)(31,38,45)(32,39,46)(33,40,47) (34,41,48)(35,42,49) (order 3)
	a_5 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30) (13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39) (28,46)(34,40)(35,47)(42,48) (order 2)
	a_6 = (6,7)(13,14)(20,21)(27,28)(34,35)(36,43)(37,44)(38,45) (39,46)(40,47)(41,49)(42,48) (order 2)

This generating set is not minimal. There is a smaller generating set with 2 elements. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup D = [G, G] is a perfect group of order 6350400, generated by {(1,27,46,19,29,41,11,5,22,48,18,33,36,13,4,26,43,20,32,40,8,6,25,47,15,34,39,12)(2,24,44,17,30, 38,9,3,23,45,16,31,37,10)(7,28,49,21,35,42,14)(1,33,41,7,31,36,5,34,42,3,29,40,6,35,38)(2,30,37) (4,32,39)(8,19,27,14,17,22,12,20,28,10,15,26,13,21,24)(9,16,23)(11,18,25)(43,47,48,49,45)} and $G/D \cong C_2^2$.

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	12700800	---	cyclic	(1,8,13,48,46,4)(2,36,9,41,44,39)(3,15,14,34,47,25) (5,22,10,20,49,32)(6,43,11)(7,29,12,27,45,18) (16,42,30,40,23,38)(17,21,35,33,26,24)(19,28,31) (1,10,40,23,15,14,33,27)(2,17,42,30,20,7,31,41) (3,38,37,16,21,35,34,6)(4,45,39,44,18,49,32,48) (5,24,36,9,19,28,29,13)(8,12,26,22)(11,47,25,43)
2	12700800	---	cyclic	(1,25,12,21,30,6,22,11,19,35,2,27,8,18,33,7,23,13, 15,32,5,28,9,20,29,4,26,14,16,34)(3,24,10,17,31) (36,46,40,49,37,48)(38,45)(39,47,42,44,41,43) (1,46,31,9,40,22,18,3,44,33,8,39,24,16,5,43,32, 10,37,26,15,4,45,30,12,36,25,17,2,47,29,11,38, 23,19)(6,48,34,13,41,27,20)(7,49,35,14,42,28,21)
3	12700800	---	cyclic	(1,14,43,21)(2,9,44,16)(3,10,45,17)(4,12,46,19) (5,11,47,18)(6,13,48,20)(7,8,49,15)(22,35) (23,30)(24,31)(25,33)(26,32)(27,34)(28,29) (36,42)(39,40)(1,38,32,26)(2,17,30,19)(3,31,33,5) (4,24,29,40)(6,10,34,12)(7,45,35,47)(8,41,11,27) (9,20)(14,48)

Sylow subgroups							
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
						(6,7)(13,14)(20,21)(22,43)(23,44)(24,45) (25,46)(26,47)(27,49)(28,48)(34,35)(41,42)

2	256	99225	nilpotent	256	*G*	(8,43)(9,44)(10,45)(11,46)(12,47)(13,48)(14,49)(22,36)(23,37)(24,38)(25,39)(26,40)(27,41)(28,42), (15,29)(16,30)(17,31)(18,32)(19,33)(20,34)(21,35)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(2,8)(3,36)(4,15)(5,29)(6,22)(7,43)(10,37)(11,16)(12,30)(13,23)(14,44)(17,39)(19,32)(20,25)(21,46)(24,41)(26,34)(28,48)(31,40)(35,47)(42,45)
3	81	4900	abelian	5184	6350400	(3,4,6)(10,11,13)(17,18,20)(24,25,27)(31,32,34)(38,39,41)(45,46,48)(8,29,43)(9,30,44)(10,31,45)(11,32,46)(12,33,47)(13,34,48)(14,35,49), (2,5,7)(8,43,29)(9,47,35)(10,45,31)(11,46,32)(12,49,30)(13,48,34)(14,44,33)(16,19,21)(23,26,28)(37,40,42), (15,36,22)(16,37,23)(17,38,24)(18,39,25)(19,40,26)(20,41,27)(21,42,28)
5	25	15876	abelian	1600	6350400	(15,43,29,36,22)(16,44,30,37,23)(17,45,31,38,24)(18,46,32,39,25)(19,47,33,40,26)(20,48,34,41,27)(21,49,35,42,28)(3,4,6,7,5)(10,11,13,14,12)(17,18,20,21,19)(24,25,27,28,26)(31,32,34,35,33)(38,39,41,42,40)(45,46,48,49,47)
7	49	14400	abelian	1764	6350400	(1,6,2,5,3,4,7)(8,13,9,12,10,11,14)(15,20,16,19,17,18,21)(22,27,23,26,24,25,28)(29,34,30,33,31,32,35)(36,41,37,40,38,39,42)(43,48,44,47,45,46,49)(1,49,32,17,26,37,13)(2,48,29,21,25,38,12)(3,47,30,20,22,42,11)(4,45,33,16,27,36,14)(5,44,34,15,28,39,10)(6,43,35,18,24,40,9)(7,46,31,19,23,41,8)

37. Let *G* be the primitive group *A*7 on 1-sets[2.3]. We have $|G| = 25401600 = 2^8 \times 3^4 \times 5^2 \times 7^2$.

Generators of *G*:	a_1 = (1,2,3,4,5,6,7)(8,9,10,11,12,13,14)(15,16,17,18,19,20,21)(22,23,24,25,26,27,28)(29,30,31,32,33,34,35)(36,37,38,39,40,41,42)(43,44,45,46,47,48,49) (order 7)
	a_2 = (1,8,15,22,29,36,43)(2,9,16,23,30,37,44)(3,10,17,24,31,38,45)(4,11,18,25,32,39,46)(5,12,19,26,33,40,47)(6,13,20,27,34,41,48)(7,14,21,28,35,42,49) (order 7)
	a_3 = (5,6,7)(12,13,14)(19,20,21)(26,27,28)(33,34,35)(40,41,42)(47,48,49) (order 3)
	a_4 = (29,36,43)(30,37,44)(31,38,45)(32,39,46)(33,40,47)(34,41,48)(35,42,49) (order 3)
	a_5 = (2,8)(3,15)(4,22)(5,29)(6,36,7,43)(10,16)(11,23)(12,30)(13,37,14,44)(18,24)(19,31)(20,38,21,45)(26,32)(27,39,28,46)(34,40,35,47)(41,42,49,48) (order 4)
	a_6 = (6,7)(13,14)(20,21)(27,28)(34,35)(36,43)(37,44)(38,45)(39,46)(40,47)(41,49)(42,48) (order 2)

This generating set is not minimal. There is a smaller generating set with 2 elements. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup *D* = [*G*, *G*] is a perfect group of order 6350400, generated by {(1,28,18,33,36,49,11,5,22,21,32,40,43,14,4,26,15,35,39,47,8,7,25,19,29,42,46,12)(2,24,16,31,37,45,9,3,23,17,30,38,44,10)(6,27,20,34,41,48,13)(1,33,28,48,3,29,26,49,6,31,22,47,7,34,24,43,5,35, 27,45)(2,30,23,44)(4,32,25,46)(8,19,14,20,10,15,12,21,13,17)(9,16)(11,18)(36,40,42,41,38)} and $G/D \cong C_4$.

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	6350400	perfect	cyclic	C_4

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	12700800	---	cyclic	(1,12,32,17)(2,14,30,21)(3,8,33,18)(4,10,29,19)(5,11,31,15)(6,13,34,20)(7,9,35,16)(22,47,25,45)(23,49)(24,43,26,46)(27,48)(28,44)(36,40,39,38)(37,42)(1,16,11,5,17,13)(2,18,12,3,20,8)(4,19,10,6,15,9)(7,21,14)(22,44,39,33,24,48,36,30,25,47,38,34)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	256	99225	nilpotent	256	*G*	(4,5)(11,12)(15,22,36,43)(16,23,37,44)(17,24,38,45)(18,26,39,47)(19,25,40,46)(20,27,41,48)(21,28,42,49)(32,33)(2,8,6,29)(3,15)(4,43)(5,22)(7,36)(9,13,34,30)(10,20,31,16)(11,48,32,44)(12,27,33,23)(14,41,35,37)(18,45)(19,24)(21,38)(25,47)(28,40)(39,49)(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)
3	81	4900	abelian	5184	6350400	(4,7,5)(11,14,12)(18,21,19)(25,28,26)(32,35,33)(39,42,40)(46,49,47)(2,3,6)(9,10,13)(16,17,20)(23,24,27)(30,31,34)(37,38,41)(44,45,48)(15,43,36)(16,44,37)(17,45,38)(18,46,39)(19,47,40)(20,48,41)(21,49,42)(8,22,29)(9,23,30)(10,24,31)(11,25,32)(12,26,33)(13,27,34)(14,28,35)
5	25	15876	abelian	1600	6350400	(15,29,43,22,36)(16,30,44,23,37)(17,31,45,24,38)(18,32,46,25,39)(19,33,47,26,40)(20,34,48,27,41)(21,35,49,28,42)(3,6,7,4,5)(10,13,14,11,12)(17,20,21,18,19)(24,27,28,25,26)(31,34,35,32,33)(38,41,42,39,40)(45,48,49,46,47)
7	49	14400	abelian	1764	6350400	(1,7,3,4,2,6,5)(8,14,10,11,9,13,12)(15,21,17,18,16,20,19)(22,28,24,25,23,27,26)(29,35,31,32,30,34,33)(36,42,38,39,37,41,40)(43,49,45,46,44,48,47)(1,36,43,29,15,22,8)(2,37,44,30,16,23,9)(3,38,45,31,17,24,10)(4,39,46,32,18,25,11)(5,40,47,33,19,26,12)(6,41,48,34,20,27,13)(7,42,49,35,21,28,14)

38. Let G be the primitive group $A7$ on 1-sets[2.4]. We have $|G| = 50803200 = 2^9\times3^4\times5^2\times7^2$.

	a_1 = (1,2,3,4,5,6,7)(8,9,10,11,12,13,14)(15,16,17,18,19,20,21)(22,23,24,25,26,27,28)(29,30,31,32,33,34,35)(36,37,38,39,40,41,42)(43,44,45,46,47,48,49) (order 7)

Generators of G:	a_2 = (1,8,15,22,29,36,43)(2,9,16,23,30,37,44)(3,10,17,24,31,38,45)(4,11,18,25,32,39,46)(5,12,19,26,33,40,47)(6,13,20,27,34,41,48)(7,14,21,28,35,42,49) (order 7)
	a_3 = (5,6,7)(12,13,14)(19,20,21)(26,27,28)(33,34,35)(40,41,42)(47,48,49) (order 3)
	a_4 = (29,36,43)(30,37,44)(31,38,45)(32,39,46)(33,40,47)(34,41,48)(35,42,49) (order 3)
	a_5 = (36,43)(37,44)(38,45)(39,46)(40,47)(41,48)(42,49) (order 2)
	a_6 = (6,7)(13,14)(20,21)(27,28)(34,35)(41,42)(48,49) (order 2)
	a_7 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)

This generating set is not minimal. There is a smaller generating set with 2 elements. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup D = [G, G] is of order 12700800, generated by {(1,18,29,4,15,32)(2,20,30,6,16,34)(3,17,31)(5,21,33,7,19,35)(8,39,43,25)(9,41,44,27)(10,38,45,24)(11,36,46,22)(12,42,47,28)(13,37,48,23)(14,40,49,26)(1,38,4,42)(2,40)(3,39,7,36)(5,37)(6,41)(8,10,11,14)(9,12)(15,24,46,35)(16,26,44,33)(17,25,49,29)(18,28,43,31)(19,23,47,30)(20,27,48,34)(21,22,45,32)} and $G/D \cong C_2^2$.

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	25401600	---	cyclic	(1,5,33,34,27,25,39,42,14,10,45,43)(2,19,30,20,23,18,37,21,9,17,44,15)(3,47,29,6,26,32,41,28,11,38,49,8)(4,40,35,13,24,46,36,7,12,31,48,22)(1,40,45,18,29,37,3,19,43,39,31,16)(2,5,47,46,32,30)(4,33,44)(6,12,48,11,34,9)(7,26,49,25,35,23)(8,41,10,20)(14,27)(15,36,38,17)(21,22,42,24)
2	25401600	---	cyclic	(1,5,47,44,30,29)(2,33,43)(3,12,45,9,31,8)(4,40,48,16,35,22)(6,19,49,23,32,36)(7,26,46,37,34,15)(11,38,13,17,14,24)(18,42,27)(20,21,28,25,39,41)(1,14,3,35,2,49,4,28,6,21)(5,42)(8,10,31,30,44,46,25,27,20,15)(9,45,32,23,48,18,22,13,17,29)(11,24,34,16,43)(12,38,33,37,47,39,26,41,19,36)
3	25401600	---	cyclic	(1,16,45,36,2,17,43,37,3,15,44,38)(4,20,46,41)(5,21,47,42)(6,18,48,39)(7,19,49,40)(8,9,10)(11,13)(12,14)(22,23,24)(25,27)(26,28)(29,30,31)(32,34)(33,35)(1,26,14,30,6,24,8,33,7,23,13,31)(2,27,10,29,5,28,9,34,3,22,12,35)(4,25,11,32)(15,40,21,37,20,38)(16,41,17,36,19,42)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
						(36,43)(37,44)(38,45)(39,46)(40,47)(41,48)(42,49)(2,8)(3,22)(4,15)(5,29)(6,36)(7,43)(10,23)(11,16)(12,30)(13,37)(14,44)(17,25)(19,32)(20,39)(21,46)(26,31)

2	512	99225	nilpotent	512	*G*	(27,38)(28,45)(34,40)(35,47)(42,48)(2,3)(4,5)(9,10)(11,12)(16,17)(18,19)(23,24)(25,26)(30,31)(32,33)(37,38)(39,40)(44,45)(46,47)(2,4)(9,11)(16,18)(23,25)(30,32)(37,39)(44,46)
3	81	4900	abelian	10368	6350400	(5,7,6)(12,14,13)(19,21,20)(26,28,27)(33,35,34)(40,42,41)(47,49,48)(2,4,3)(9,11,10)(16,18,17)(23,25,24)(30,32,31)(37,39,38)(44,46,45), (22,36,43)(23,37,44)(24,38,45)(25,39,46)(26,40,47)(27,41,48)(28,42,49)(8,15,29)(9,16,30)(10,17,31)(11,18,32)(12,19,33)(13,20,34)(14,21,35)
5	25	15876	abelian	3200	6350400	(15,22,36,43,29)(16,23,37,44,30)(17,24,38,45,31)(18,25,39,46,32)(19,26,40,47,33)(20,27,41,48,34)(21,28,42,49,35)(3,6,4,7,5)(10,13,11,14,12)(17,20,18,21,19)(24,27,25,28,26)(31,34,32,35,33)(38,41,39,42,40)(45,48,46,49,47)
7	49	14400	abelian	3528	6350400	(1,2,3,6,5,7,4)(8,9,10,13,12,14,11)(15,16,17,20,19,21,18)(22,23,24,27,26,28,25)(29,30,31,34,33,35,32)(36,37,38,41,40,42,39)(43,44,45,48,47,49,46)(1,30,45,41,26,21,11)(2,31,48,40,28,18,8)(3,34,47,42,25,15,9)(4,29,44,38,27,19,14)(5,35,46,36,23,17,13)(6,33,49,39,22,16,10)(7,32,43,37,24,20,12)

8. DISCUSSION

In this article the problem of cigarette combustion globally is considered. The research showed that this problem is neither easy nor simple, just opposite it is a very hard scientific problem, which includes other complex subproblems, as those of problem of balancing chemical reaction of tobacco combustion, field temperature problem in the combustion zone, smoke filtration problem, and the problem of groups' formation of reaction coefficients. No one of the mentioned subproblems is fully solved. Their solutions are quite tied with usage of update hardware and software. For instance, with current softwear it is imposible to be balanced the chemical reaction (4. 28).

Now, this question naturally arises: *Is it solvable any chemical equation by a computer*?

The reply of this question is very simple: No! Reason why this reply is negative lies in inappropriate chemical methods, which most of them are paradoxal, but from other hand sophisticated mathematical methods are not computer adapted for daily usage. Right now, the hardest problem in chemistry as well as mathematics is balancing of $\aleph$ reactions, *i.e.*, the continuum reactions, which need special treatment with new mathematical methods, as this developed in this article.

Since the chemical reaction of tobacco combustion (4. 28) is very complex reaction, which belongs to the class of $\aleph$ reactions, its necessary and sufficient conditions are not determined. In fact, it is a shortcoming of the mathematical model, which does not provide precise necessary and sufficient conditions when the reaction (4. 28) is possible.

Of course, there are other obstacles, which must be overcome. For instace, for the reaction (4. 28) is not developed topology of its solutions.

These remarks creates a new scientific requirement that for balancing $\aleph$ reactions should be built a computer package as soon as possible, because this kind of chemical reactions is not studied enough in chemistry and mathematics too.

Generaly speaking it is a first work where the chemical reaction (4. 28) is considered, and as every pioneering job it does not provide a complete treatmant, but it opens doors of next research.

9. CONCLUSION

The global cigarette combustion problem in this work is treated only from the mathematical point of view, for whom is given a completely new approach toward its solution. It is considered as a complex mathematical problem, which includes few subproblems: the problem of balancing chemical reaction of tobacco combustion, field temperature problem in the combustion zone, smoke filtration problem, and the problem of groups' formation of reaction coefficients. In fact, these problems are particularly solved for the certain simulation conditions. For instance, the smoke infiltration problem is founded and solved by virtue of partial differential equation of first order. The field temperature problem in the combustion zone is modeled by the two-dimensional heat transfer equation which is solved by quadratures. The chemical reaction (4. 28), which describes tobacco combustion is a completely new reaction and it includes all important alkaloids and toxins. This reaction has four generators. In fact, it is a very hard chemical reaction, which cannot be balanced by a computer, because right now in the theory of computer sciences there is not powerful software, which can be used for its balance. Unique way to balance this reaction is by the usage of mathematical method.

Since, the reaction of tobacco combustion is very complicated we found only its general solution and one particular solution. This reaction spans real vector spaces. For the reaction coefficients are calculated a symmetric group S_{49}, an alternating group A_{49} and 38 primitive groups.

The strengths of the mathematical model are:

1. This model provides an alternative approach for balancing ℵ chemical reactions.
2. Since this model is well formalized, it belongs to the class of consistent models for balancing chemical reaction.
3. This model showed that any chemical reactions can be treated as *n*-dimensional geometrical entity.
4. In fact, here-offered model simplifies mathematical operations provided by the previous well-known matrix methods and is very easily acceptable for daily practice. The model has this advantage, because it fits for all ℵ chemical reactions, which previously were balanced only by the methods of generalized matrix inverses.
5. The mathematical model provides the dimension of the solution space.
6. Also, by this method a basis of the solution space is determined.
7. This method gives an opportunity to be extended with other numerical calculations necessary for ℵ reactions.
8. It can predict quantitative relations among reaction coefficients.
9. The mathematical model can predict reaction stability.
10. Offered mathematical model represents a well basis for building a software package.

The weak sides of the mathematical model are:

1. By this model the minimal reaction coefficients cannot be determined.
2. Also, this model cannot recognize when chemical reaction reduces to one generator reaction.
3. This model cannot arrange molecules disposition.
4. It does not provide precise necessary and sufficient conditions when the ℵ reaction (4. 28) is possible.
5. This model cannot generate topology of reaction.

This model wild opens the doors in chemistry and mathematics too, for a new research of ℵ chemical reactions, which unfortunately today cannot be balanced by usage of computer, because there is not such method. Here developed mathematical model is a big challenge for researchers to extend and adapt it for a computer application. Sure that it is not easy and simple job, but it deserves to be realized as soon as possible.

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SYMBOLS

α – fraction, $(0 \le \alpha \le 1)$,
β – tobacco absorption coefficient, 1/mm

C_i – concentration of components X_i, ($1 \leq i \leq n$), mg/mm
L – total cigarette length, mm
L_0 – initial length of the cigarette, mm
ΔL – burning cigarette length, mm
x – distance from the burning end, mm
y – coordinate, mm
t – time, s
T– temperature, ºC
k – tobacco thermal diffusivity, m^2/s.

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